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A Coupled Model of Population, Poaching, and Economic Dynamics to Assess Rhino Conservation Through Legal Trade

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Abstract
Rhinoceros populations in Africa are in peril largely due to the high value of their horns and the poaching that ensues. The strategy of legalizing the international trade of rhino horn is receiving increased support among both the people and government officials in Africa. Many in the international conservation community remain opposed to the idea. The legalization strategy is straightforward in theory: legalizing the trade of rhino horn will introduce a large quantity of horn to the market, the increased supply will lead to lower prices for rhino horn, and lower prices will reduce the overall poaching pressure these animals face. In this work, we propose a model for rhino populations that includes the interrelated dynamics of the price of rhino horn and poaching rates to establish thresholds of parameter values for which legalization can either increase or decrease rhino populations.

Keywords: mathematical biology, conservation, rhinoceros, poaching, dynamical systems

1 Introduction

The African rhino, whether it be black or white, has been a staple of culture in the savannah/bush landscapes of Africa, with cave paintings dating back thousands of years demonstrating their presence. While native people treated the powerful rhinos with respect and even hesitancy, their size no longer protects rhinos against modern technology and economic forces. In recent decades, rhino populations across Africa have decreased dramatically. The northern white rhino (Ceratotherium simum cottoni) went functionally extinct in 2018. From 2018–2022, southern white rhino (Ceratotherium simum simum) numbers have declined from 18,000 to 16,000, a 12% drop [8]. From 1973 to 1994, black rhino (Diceros bicornis) populations decreased by 96% to a low of 2,300. Through massive conservation efforts, the black rhino population now stands at a still-low 6,500 [8].

The primary cause of the steep population decline is increased poaching fueled by the international demand for rhino horns. While composed of keratin, the same protein found in human hair and fingernails, rhino horns have been used in East and Southeast Asian traditional medicine for centuries to treat ailments ranging from fever to gout, to erectile dysfunction [4]. In addition, horns are used ceremonially, with pieces displayed as status symbols in homes and offices, as well as on the back of knives for example.

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To combat the decline in rhino numbers, world organizations such as CITES (The Convention of International Trade in Endangered Species of Wild Fauna and Flora) have banned all rhino trade globally. With legal trade banned, the black market has driven the price of rhino horns to incredible levels. The current price of raw horn in USD is estimated to be $20,000 per kilogram [5], a price that is higher than that of gold, and even many illegal drugs like cocaine. As horns have an average mass of 4 kilograms, there is a tremendous financial incentive for poachers to kill rhinos.

As the label of extinction draws closer despite the ban, economists and conservationists have begun to consider alternative approaches. Many, particularly those from African countries that have rhinos, believe legalizing the international trade of rhino horn will reduce horn prices, which will lead to a decrease in poaching, and therefore increase rhino viability and conservation [1]. Proponents of the legal trade cite examples like the crocodile, where the legalization of the leather trade ultimately led to an increase in population numbers. One source of the legal trade would be the horns of rhinos that have died naturally. In fact, a large stockpile of rhino horn has already been collected in this way in Africa. Another source of legal rhino horn could come from cutting off the horns of live animals, both in the wild and on farms [5]. Like human fingernails, rhino horn is made of keratin and will grow back after being cut. Each year the horn of a rhino grows about 3 to 4 inches which means that as rhino populations recover, the supply of horn could theoretically become a renewable resource.

Those opposed to legalizing the trade of rhino horns point to the possibility that legalization could dramatically increase the demand for rhino horns. For starters, the lower horn prices that result from legalization could increase demand for horns so much that poaching actually increases. In addition, there is also concern that legalized trade may unlock new demand as people who would not illegally buy rhino horn may become willing to do so. This means the market will not only have the increased demand of the original black market customers due to lower horn prices, but will also include the demand of new consumers who are now willing to buy it because it is no longer illegal.

Given the high stakes, the issue has received attention in the modeling literature. An early model used optimal control to establish the frequency with which rhino horns could be harvested from live animals [11]. The framework of predator-prey dynamics has been used to model the interaction between poachers and rhinos [10]. Game theory has been used to investigate the competing strategies of poachers and wildlife managers [7, 9].

In this paper, we investigate the original principles behind the argument to legalize the trade of rhino horns: that the availability of legal horn on the market will reduce its price and therefore reduce the incentive to poach rhinos. While simple in theory, the ultimate result of such a strategy depends on the quantitative details: "How much legal horn?", "How much will the price decrease?", and "How sensitive will poaching intensity be to price?" As exact answers to these questions are not available ahead of time, we create a model built on accessible mathematical ideas that can be used to evaluate assumptions on how legalization could play out. The model's modular design means that stakeholders from various backgrounds can adjust the framework to their assumptions and investigate the consequences.

2 Mathematical Model

In this section, we create a mathematical model for the population of African rhinos that includes the interrelated dynamics of the price of rhino horn and poaching rates. The dynamics of the rhino population are established in Section 2.1 and the price dynamics of rhino horn are described in Section 2.2.

2.1 Rhino population model

To model the population of African rhinos, we use the differential equation given by

$$\frac{dR}{dt} = (\text{logistic growth rate}) - (\text{poaching rate})$$

or

$$\frac{dR}{dt} = rR\left(\frac{K-R}{K}\right) - \left(\frac{k_{\max}}{1+be^{-vp}}\right)R, \qquad (1)$$

where $R(t)$ represents the number of rhinos in Africa at time t and

$$\begin{aligned} r &= \text{exponential growth rate of rhino population,} \\ K &= \text{carrying capacity for rhinos in Africa,} \\ p &= \text{price of rhino horn in USD per kilogram,} \\ k_{\max} &= \text{maximum poaching intensity (prop killed per year),} \\ v &= \text{sensitivity of poaching rate to rhino price,} \\ b &= \text{logistic parameter that determines an initial value.} \end{aligned}$$

The first term in Equation (1) describes the logistic growth rate of the rhino population in the absence of poaching. When the population is low, the population grows at a rate that is proportional to the size of the population (i.e. exponential growth with rate r). As the population increases, resource limitations slow the growth as the population approaches K, the carrying capacity for rhinos in Africa.

The graph of our logistic population growth is illustrated in Figure 1. Here, we assume a current carrying capacity of $K = 200000$ rhinos based on estimates that there were approximately 500000 rhinos in Africa at the beginning of the 20$^{\text{th}}$ century and the realization that human development on the continent means such numbers are no longer sustainable [8]. We assume that the population grows by approximately 2% per year (i.e. $r = 0.02$), based on the growth rate exhibited by the white rhino population in the 1970s [8]. We assume the initial population of rhinos to be 25,000 (i.e. $R(0) = 25000$) [8]. The simulation in Figure 1 represents a hypothetical baseline where rhino poaching immediately stops and rhino populations are able to recover.

To model rhino poaching that depends on the price of horns, we use

$$k(p) = \frac{k_{\max}}{1+be^{-vp}} \qquad (2)$$

where $k(p)$ describes the overall "poaching intensity" (second term in Equation (1)). This quantity includes the overall effects of the number of people engaged in poaching as well as the frequency and duration of their poaching activities. We assume that poaching occurs at a rate that is proportional to the size of the rhino population (i.e. at a rate of $k(p)R$). For example, if rhino numbers are low the same poaching intensity will result in fewer killed rhinos as rhinos will be more difficult for poachers to track and locate and because anti-poaching efforts would be more effective in protecting smaller populations. If the rhino population is significantly larger, then rhinos are harder to protect and easier for poachers to locate.

In formulating the dependence of poaching intensity on the price of rhino horn, it is clear that poaching should increase as the price of horn increases. However, we assume an upper bound on poaching intensity because, at some point, prices are so high that poachers will already be putting all the effort they possibly can into poaching (and therefore not be affected by further increases in price). When prices are relatively lower, we would expect the poaching rate to be more sensitive to changes in horn price. To include these characteristics in modeling poaching intensity we use the logistic function given in Equation (2).

Current data estimates the total population of rhinos in Africa to be around 25,000 animals and that roughly 500 were poached in each of the last three years (i.e. 2% of the total population of 25000 per year) [8]. We assume a maximum poaching intensity of 8% per year (i.e. $k_{max} = 0.08$) and a value of $v = 0.00009$ as this produces a realistic sensitivity to horn price. As rhino horn currently sells for about 20,000 USD/kg [5], we require that our model satisfies $k(20000) = 0.02$ which produces a value of $b = 18.15$. A graph of this resulting function is shown in Figure 2.

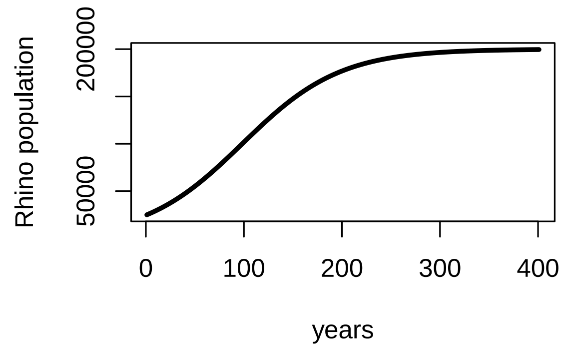


Figure 1: The logistic model for the population of rhinos in Africa without poaching (i.e. solution to Equation (1) with $K = 200000, k = 0.02, R(0) = 25000$, and $k_{\max} = 0$).

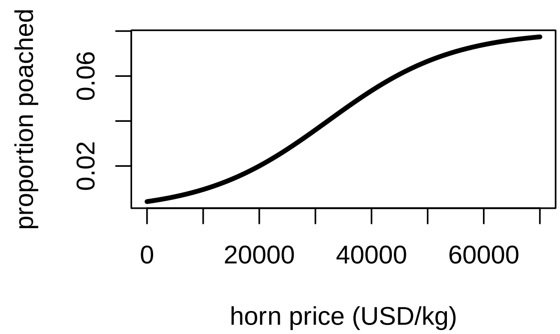


Figure 2: Poaching intensity as a function of rhino horn price assuming a poaching intensity of 2% when horn price is 20000 USD/kg and $k_{max} = 0.08, v = 0.00009$.

2.2 Horn price model

Supply and demand are two fundamental economic principles describing how changes in the price of a resource, commodity, or product affect its supply and demand. In introductory economics courses, supply and demand are often visualized as linear functions as shown in Figure 3. As price increases, the supply of that good will rise as represented by the blue curve in Figure 3. The red curve in Figure 3 represents demand and illustrates the situation that as price increases, the demand for a good will decrease. As the price drops, supply condenses while demand grows.

In general, if the price of a good is at a point where supply is bigger than the demand, the price will decrease. This decrease in price will in turn affect levels of supply and demand. Specifically, as price goes down, demand will increase and supply will decrease. In Figure 3, this is the case if the starting price is 8 (shown with green dashed line). As we can see, price continues to decrease until supply and demand are equal which happens at a price of slightly less than 6.

If the price of a good begins at a value for which supply is less than demand, the price of that good will increase. This increase in price will then decrease demand but also increase supply. Again this continues until supply and demand are equal. In Figure 3, we see the example in green dashed lines where the initial price is 3.

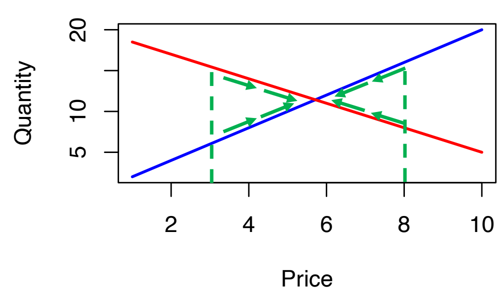


Figure 3: A graph of supply and demand for an arbitrary product. Supply increases as price increases (blue line) and demand decreases as price increases (red line).

As we see in Figure 3 regardless of the starting value, price will adjust to a point where supply and demand are equal. This is the process of discovering the price on the

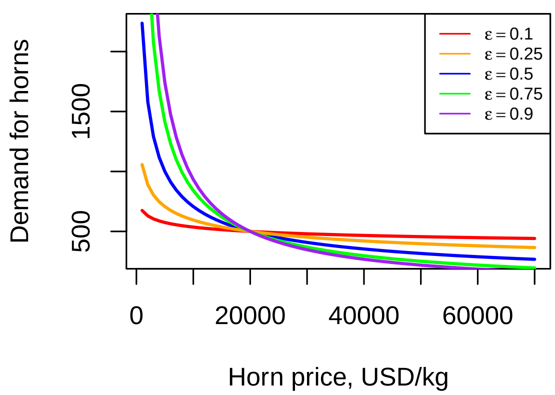


Figure 4: The demand for rhino horns as a function of price (i.e. Equation (3)) with varying values of demand elasticity to price (i.e. values of ε) assuming an initial demand of 500 rhino horns when the price is 20000 USD/kg and no expansion of demand due to legalization (i.e. $f = 1$).

open market. This equilibrium price is what is referred to as the "market price" and is the price the good will actually cost on the open market.

While linear functions are appropriate for introducing the ideas of supply and demand, they are not realistic in most situations. To model the demand for rhino horn as a function of price, we used the formula of [2] given by

$$\text{demand} = 500f\left(\frac{20000}{p}\right)^{\varepsilon}. \tag{3}$$

Here, p represents the price of the rhino horn in USD/kg, ε represents the elasticity of demand to the price of rhino horn, and f is a factor of expanded demand for horn due to legalization. Notably, we assume a current price for rhino horn of 20000 USD/kg and that the horns of 500 poached rhinos will be demanded at that price. The graph of this demand curve is shown in Figure 4.

As shown in Figure 4, demand for rhino horn decreases as the price increases. While we use the value of $\varepsilon = 0.50$ as a baseline following the work of [2], Figure 6 shows demand curves for various values of price elasticity. For example, if $\varepsilon = .10$ (i.e. if price elasticity is very low), we see that the demand for rhino horn barely changes regardless of what the price is (see red curve). If price elasticity is high (e.g. $\varepsilon = 0.90$, purple curve), we see that demand varies drastically depending on the price.

To model the supply of rhino horn, we use

$$\text{supply} = \left(\frac{k_{\max}}{1+be^{-vp}}\right)R + h_{\text{legal}}, \tag{4}$$

which includes horns supplied through poaching and through the hypothetical legal trade of horns. Notably, we quantify supply and demand in terms of the number of horns even though horn size varies and trade often involves partial horns. Doing so allows the model to directly link the number of poached rhinos in Equation (1) to the supply of rhino horns in the first term in Equation (4). As the details of how legalization would be implemented are yet to be determined, we make the simplifying assumption that a constant number of legal rhino horns will be put on the market each year, which we denote h_{legal}.

We combine the above considerations and assume that the price of rhino horn will change at a rate that is proportion to the difference between supply and demand to arrive at our model of the price dynamics for the trade of rhino horn. The model is given by

$$\begin{aligned}\frac{dp}{dt} &= -s\,(\text{supply} - \text{demand})\\ &= -s\,(\text{poached horn} + \text{legal horn supply} - \text{demand})\\ &= -s\left[\left(\frac{k_{\max}}{1+be^{-vp}}\right)R + h_{\text{legal}} - 500f\left(\frac{20000}{p}\right)^{\varepsilon}\right].\end{aligned}$$

with parameters

$$\begin{aligned}\varepsilon &= \text{price elasticity of demand for rhino horn,}\\ s &= \text{rate of price adjustment,}\\ h_{\text{legal}} &= \text{amount of legal horns on market per year,}\\ f &= \text{factor of expanded demand due to legalization.}\end{aligned}$$

We use a value of $s = 0.80$ to reflect the fact that the dynamics of horn price will progress much more quickly than those of the rhino population.

2.3 Full model

Putting together our formulations for the dynamics of the rhino population and price of rhino horn, we arrive at our full model:

$$\begin{aligned}\frac{dR}{dt} &= rR\left(\frac{K-R}{K}\right) - \left(\frac{k_{\max}}{1+be^{-vp}}\right)R,\\ \frac{dp}{dt} &= -s\left[\left(\frac{k_{\max}}{1+be^{-vp}}\right)R + h_{\text{legal}} - 500f\left(\frac{20000}{p}\right)^{\varepsilon}\right],\end{aligned} \tag{5}$$

with state variables given by

$$\begin{aligned}R(t) &= \text{ the number of rhinos in Africa at time } t,\\ p(t) &= \text{ the price of rhino horn in USD/kg at time } t.\end{aligned}$$

The coupled model formalizes the interrelated dynamics of the rhino population, poaching intensity, and horn price. The complete list of the parameters of the model as well as the values/ranges we use for each is given in Table 1.

In the next section, we fix the values of the parameters K, r, k_{max}, v, b, and s for all simulations. To focus on

Table 1: Summary of parameters for the full model.

	Description	Value	Source
K	carrying capacity for rhinos in Africa	200000	assumption related to [8]
r	exponential growth rate of rhino population	0.02	assumption related to [8]
k_{max}	maximum rate that poachers can kill rhinos	0.08	assumption
v	sensitivity of poaching rate to rhino price	0.0009	assumption
b	parameter that determines initial poaching intensity	18.15	calibrated
s	rate price adjusts to difference between supply and demand	0.80	assumption
ε	price elasticity of demand for rhino horn	0.50, 0.10–0.90	[2], exploratory variable
h_{legal}	rate that legal supply of rhino horn is put on market	0–1000	exploratory variable
f	factor of expanded demand for horn due to legalization	1–2	exploratory variable

the potential consequences of legalizing the trade of rhino horn, our analysis will investigate ranges of values for the parameters most directly related to legalization; namely, ε, price elasticity of demand for rhino horn, h_{legal}, the amount of legal horns put on the market per year, and f, the factor of expanded demand for rhino horn due to legalization.

3 Results

To analyze the potential outcomes of legalizing the trade of rhino horn, we begin by establishing a baseline to compare against. Figure 5 shows the solution of our full model given in System (5) assuming our baseline parameter values from Table 1 and no legal horn trade (i.e. h_{legal} and $f = 1$). This represents the viability of the rhino population over the next 80 years if the rhino horn trade continues to remain illegal. The first graph in Figure 5 represents rhino population numbers over a period of 80 years. The graph demonstrates a decline in rhino population numbers, with the original 25,000 rhinos at year zero becoming 16,000 after 80 years. The second graph in Figure 5 shows the number of poached rhinos per year over the same 80-year period. The number of poached rhinos per year declines from 500 to 390 over this time period due to the fact that the rhino population decreases significantly. The third graph in Figure 5 shows the price of horns rising from 20,000 USD/kg to about 24,000 USD/kg which occurs as the reduced numbers of poached rhinos result in a reduced supply of rhino horns.

Figure 6 shows the simulation results of System (5) when the legalization of the horn trade is conisdered. To do so, the variable of h_{legal} representing the number of legal horns put on the market per year is introduced. The simulation result depicted in red for $h_{\text{legal}} = 0$ is identical to the baseline simulation in Figure 5. At this point, we assume no expansion of demand due to the legalization of trade (i.e. $f = 1$). The first graph in Figure 6 shows the rhino population assuming various levels of h_{legal} over the 80 years. We see a direct relationship between the supply of legal horns and population numbers with more legal horns leading to more rhinos. For example, when 1000 horns per year are introduced the rhino population rises to 45,000 after 80 years in comparison to the decline to 15,000 when zero legal horns are introduced and poaching continues. The population remains relatively stable through the simulations if 250 legal horns are supplied each year. The second graph in Figure 6 shows the number of poached rhinos per year. Regardless of h_{legal}, we see that annual poaching numbers decrease initially. However, poaching numbers decrease much more quickly with higher supplies of legal horns. With higher supplies of horn, we see that the number of poached rhinos can even bottom out and subsequently begin to rise. This occurs as the price of horn stabilizes and higher rhino population numbers enable poachers to kill more animals with the same poaching effort. The third graph shows the price of rhino horn over the 80 years with the various amounts of h_{legal}. As a larger quantity of legal horn is introduced the price of rhino horn plummets. For example at 375 legal horns per year, the price drops to \$12,000 versus a price increase to \$25,000 where no horns are introduced. With the largest supply of horns (i.e. $h_{\text{legal}} = 1000$), we see price drop by about 75% in 30 years.

While comparing different values of the legal horn supply, h_{legal}, all simulations in Figure 6 use a baseline cost elasticity of $\varepsilon = 0.5$. Given that it is difficult to estimate how sensitive the behavior of rhino horn consumers will be to price, we examine the analogous results for different price elasticities in Figure 7. Here, we compare results for when consumer behavior is insensitive to price (i.e. $\varepsilon = 0.1$ in the top row) and when consumer behavior is more sensitive (i.e. $\varepsilon = 0.9$ in the bottom row) to our initial simulations ($\varepsilon = 0.5$ in the middle row, same as Figure 6). In the first column of Figure 7, we see that the qualitative behavior of the rhino population is the same

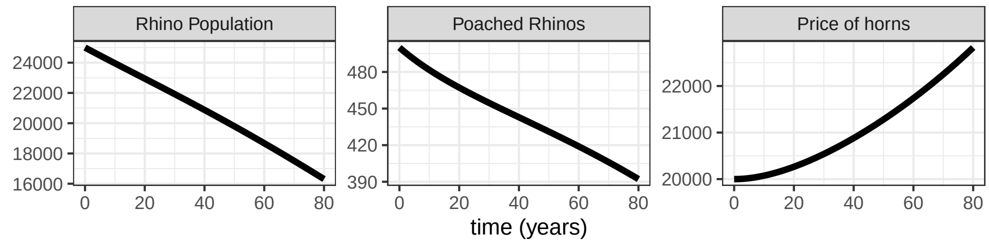


Figure 5: Baseline simulation of rhino population, annual number of poached rhinos, and the price of rhino horn over the next 80 years. The baseline scenario represents the situation where the trade of rhino horns remains illegal (i.e. $h_{\text{legal}} = 0, f = 1$). We also assume a baseline price elasticity of $\varepsilon = 0.50$.

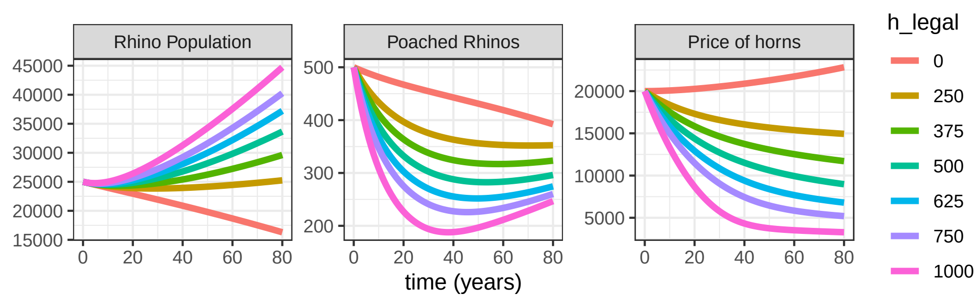


Figure 6: Simulations of rhino population, annual number of poached rhinos, and the price of rhino horn over the next 80 years when the legalization of horn trade is considered. Each curve represents a different quantity of legal horns that can be supplied per year. These simulations assume no expanded demand for rhino horn from legalization (i.e $f = 1$). We also assume a baseline price elasticity of $\varepsilon = 0.50$.

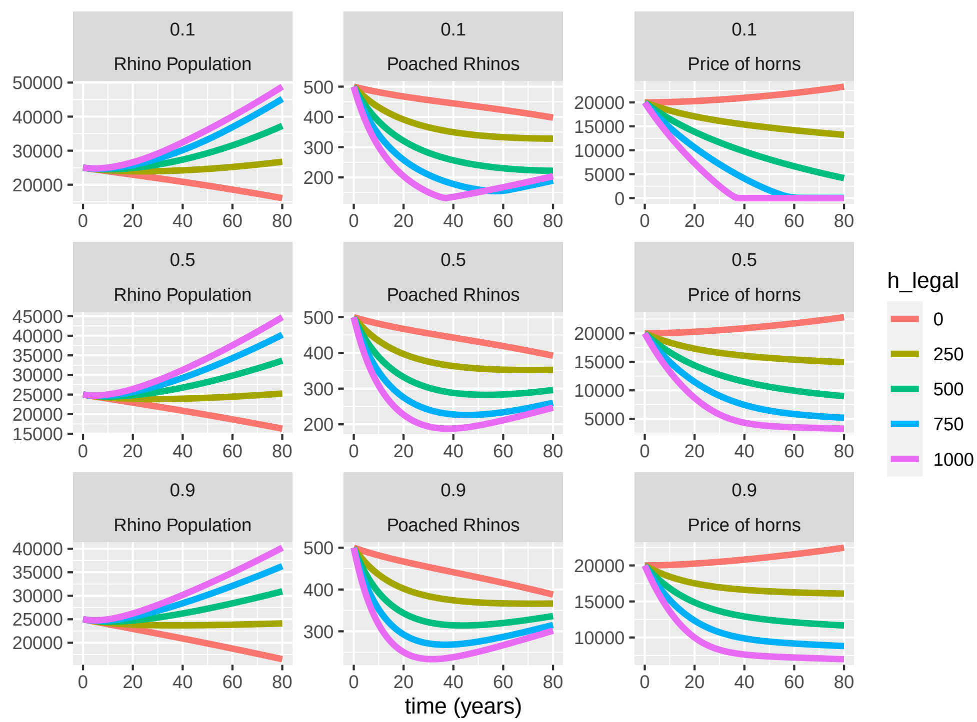


Figure 7: Simulations of rhino population, annual number of poached rhinos, and the price of rhino horn over the next 80 years for different levels of h_{legal}, when varying the values of the price elasticity to rhino horn from $\varepsilon = 0.1$, $\varepsilon = 0.5$ and $\varepsilon = 0.9$. These simulations assume no expanded demand for rhino horn from legalization (i.e. $f = 1$).

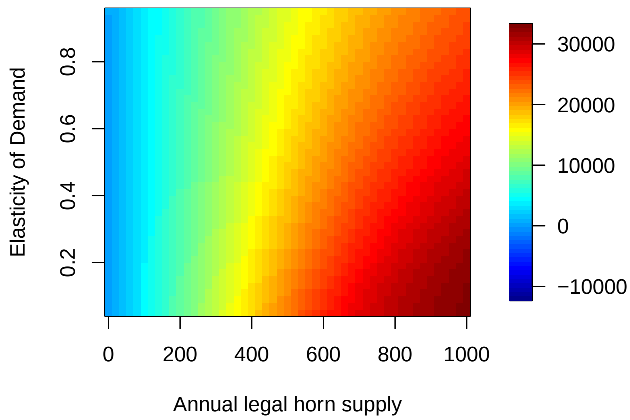


Figure 8: Rhinos saved through legalization (i.e. the number of rhinos in the population after 80 years of legalization subtract the number of rhinos after 80 years without legalization) for various combinations of legal horn supply, h_{legal}, and demand elasticity, ε. Here we assume no expansion of demand due to legalization (i.e. $f = 1$).

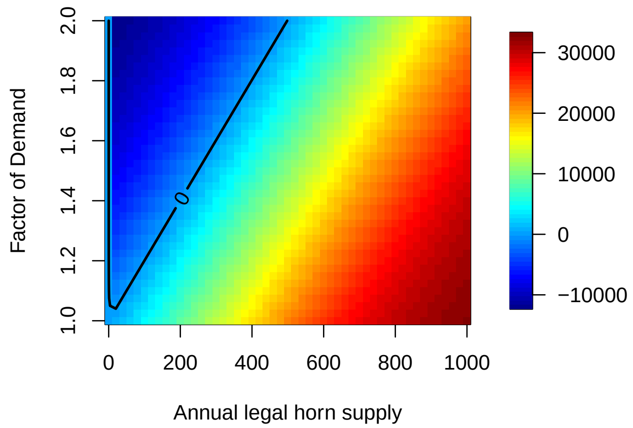
regardless of price elasticity. However, we see quantitative differences in the effect of legalization as the rhino populations reach roughly 50000, 45000, and 40000 for elasticities of $\varepsilon = 0.1, 0.5$, and 0.9, respectively. Therefore, legalization performs better when price elasticity is lower. In the third column, we see significant qualitative differences as the price of rhino horn plummets all the way to 0 in simulations with the highest supplies of legal horns and low price elasticity. As our model for poaching intensity in Equation (2) is still positive for a price of 0, we see that poaching continues even under these circumstances (see middle column).

Figure 7 only illustrates results for three values of price elasticity and five values of the legal horn supply. Given their uncertainty, we move to a more thorough examination of how the number of rhinos saved through legalization depends on the amount of legal horn that can be put on the market per year, h_{legal}, and the elasticity of demand for rhino horns, ε. To do so, we focus our attention on the difference between the number of rhinos in the population after 80 years of legalization and the number after 80 years without legalization. We refer to this as the “number of rhinos saved through legalization” but note that this number can be negative if legalization results in a lower population of rhinos after 80 years. Figure 8 shows the number of saved rhinos for h_{legal} varying from 0 to 1000 and ε from 0 to 1. In Figure 8, we see that the higher the legal horn supply and the lower the elasticity of demand, the more rhinos are saved. This can be seen when looking at elasticities of demand of $\varepsilon = 0.6$ and $\varepsilon = 0.2$ where at the same legal supply number of 600 horns there is a difference in the number of rhinos saved of about 5,000. We see in Figure 8 that the outcome of legalization is much more sensitive to the quantity of legal horn that can be supplied than to the elasticity of demand. This is observed by noticing that the contours in the surface are predominantly vertical. For example, if the horn supply is relatively low (e.g. 200 horns per year), the number of rhinos saved is about the same regardless of the elasticity of demand. The elasticity of demand only begins to have a significant effect when one assumes larger quantities of legal horn supply.

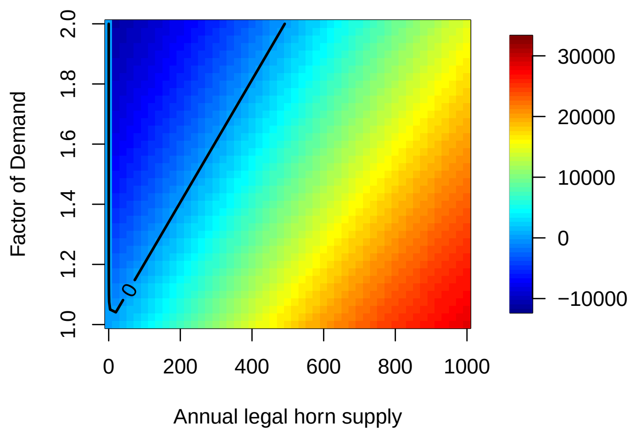
While the number of rhinos saved through legalization varies, Figure 8 shows legalization *always* resulting in a net good for the rhino population (i.e. higher populations with legalization than without). However, the results of Figure 8 omit an important aspect that opponents of legalization highlight: that legalization could unlock new demand for rhino horn as people who were unwilling to buy illegal horns become consumers of legal horns. This idea is incorporated into System (5) through the parameter, f, the factor of expanded demand for rhino horn due to legalization.

In Figure 9, we see analogous graphs to that of Figure 8 that again depict the number of rhinos saved through legalization. In Figure 9, the factor of expanded demand due to legalization varies along the vertical axis from $f = 1$ (i.e. no expanded demand) to $f = 2$ (i.e. demand that is doubled due to legalization). Values for the elasticity of demand to price are set to $\varepsilon = 0.1$, $\varepsilon = 0.5$, and $\varepsilon = 0.9$ in Figure 9a, 9b, and 9c, respectively. Figure 9 shows that the net outcome of legalization is more sensitive to the factor of expanded demand than to price elasticity as the contours are significantly more diagonal and less vertical than those of Figure 8.

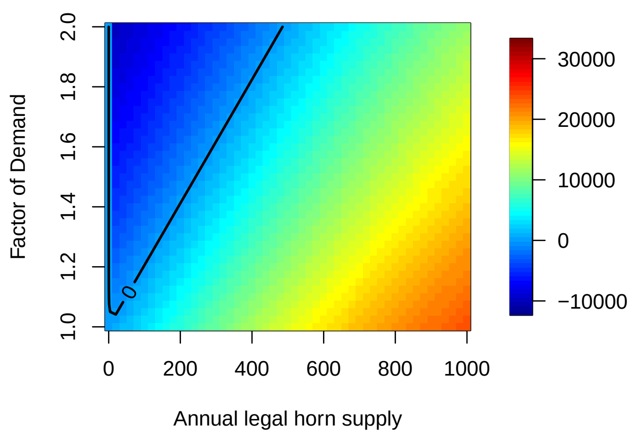
When the expansion of demand is considered, we see that it is indeed possible for legalization to have the adverse effect of reducing the population of rhinos. This occurs for lower supplies of legal horn and higher expansion of demand (i.e. upper left corner of the graph). Interestingly, we see that the contour lines representing zero rhinos saved are identical in all three graphs of Figure 9. This suggests that the threshold of whether legalization increases or decreases the rhino population is independent of the elasticity of demand. While the threshold remains the same, it is important to note that the number of rhinos saved is significantly impacted by the elasticity of demand. In the most optimistic scenario (i.e. $h_{\text{legal}} = 1000, f = 1$) for example, the number of rhinos saved over 80 years varies from under 20,000 to over 30,000. Importantly, differences in the potential negative effect of legalization are much smaller. In the case where $h_{\text{legal}} = 0$ and $f = 2$, the number of rhinos lost due to legalization varies by less than 200.



(a) elasticity of demand to price of $\varepsilon = 0.1$



(b) elasticity of demand to price of $\varepsilon = 0.5$



(c) elasticity of demand to price of $\varepsilon = 0.9$

Figure 9: Rhinos saved through legalization (i.e. the number of rhinos in the population after 80 years of legalization subtract the number of rhinos after 80 years without legalization) for various combinations of legal horn supply, h_{legal}, and expansion of demand due to legalization, f.

4 Discussion

In this work, we have created a mathematical model to investigate the potential effects that legalizing the trade of rhino horn could have on the population of rhinos in Africa. The model adds valuable insight to the debate about legalization. Importantly, we see that legalization, coupled with standard supply and demand economic dynamics, has the potential to significantly increase the rhino population in the coming decades with the rhino population in Africa increasing by as much as 30,000. However, the number of rhinos saved will be affected by the amount of legal horn that could be supplied and the behavior of the consumers who buy rhino horn.

While much of the previous debate regarding legalization has focused on issues related to demand, our modeling shows that the overall outcome of legalization is more sensitive to the quantity of legal horn supplied. Put simply, the outcomes are much better if 700 legal horns can be supplied per year versus 100 legal horns, for example. Therefore, it is vital that the proponents of legalization develop detailed plans that accurately estimate how many horns can be supplied. Without such a detailed plan, our model makes the simplifying assumption of a constant supply of legal rhino horns. While a large supply of legally obtained horns has already been amassed, the sustainability of this supply decades into the future will require harvesting horns from live animals and from naturally deceased animals. This process will of course be affected by the dynamics of the population itself (e.g. larger numbers of rhinos in the future will result in a larger supply of legal horns) but may require commitments from governments and international organizations to support legal horn harvesting until populations begin to recover.

Consumer behavior will also affect the outcome of legalization albeit to a lesser extent than the amount of legal horn supplied. If demand is insensitive to price, the potential of legalization to positively impact rhino populations is the greatest. This makes sense as the increased supply of horns will lower prices and consequently reduce the incentive to poach while only leading to a small increase in demand. If demand is more sensitive to price, the decrease in price will lead to a larger increase in demand that offsets some of the reduced incentive to poach. Overall, we find that the elasticity of demand to the price of rhino horn to have a smaller effect on the outcome of legalization than many have suggested.

Importantly, our modeling shows that legalization always leads to an increase in rhino populations regardless of the elasticity of demand under the assumption that legalization does not bring new consumers into the market. If a new consumer base does emerge, it is possible for legalization to harm the rhino population in Africa if the supply of legal horn is too small. The threshold for this effect appears to be linear and independent of the elasticity of demand to the price of rhino horns. While an analytical proof is outside the scope of this paper, a simple explanation seems to be supported in Figure 9. Specifically, if the consumer base expands by $x\%$, then the legal horn supply must provide at least $x\%$ additional horn to offset that increased demand. For example in Figure 9, we see that a 50% expansion of the consumer base (i.e. $f = 1.5$) corresponds to a threshold for h_{legal} of 250 horns per year (i.e. 50% above the baseline assumption of 500 horns a year). On the other hand, this relationship may result from our simplified formulation of the expansion of demand due to legalization (i.e. a simple demand factor, f). In reality, it is possible that this new group of consumers will represent a different type of consumer altogether (i.e. exhibiting a different elasticity of demand or a different demand curve) rather than simply an expansion of the original consumer base.

While our modeling includes specific quantities of animals, it is important to note that the goal of this work is not to make accurate numerical predictions about rhino populations several decades into the future. Instead, the value is in providing a basis for comparing different assumptions and assessing relationships between competing factors. Consequently, this modeling work does not signify a resolution to the debate about the legalization of the trade of rhino horn. As empirical studies provide more robust estimates of economic behavior and firmer plans emerge for the legalization of the trade of rhino horn, the model presented in this research can serve as a framework for more detailed future studies to investigate these and other considerations.

Author Contributions

All authors contributed to the design of the model, model description, and interpretation of the results. David Gerberry implemented the model in R for simulation studies.

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