Dynamical modelling of Kaziranga wildlife ecology

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Abstract. The ecology, conservation and management of wildlife is an important component in the fields of environmental science and natural resource management. Wildlife management is a very dynamic field. Kaziranga National Park in India has a good conservation history, due to its efficient management policies. The mathematical modelling of the ecosystem in Kaziranga national park located in Assam, India with the interaction of species between Indian Rhinoceros, Bengal Tiger and Swamp deer has been studied. In this system, predator-prey interaction and the effect of poaching for both Indian Rhinoceros and Bengal Tiger have been considered. Also, the indirect dependency between Indian Rhinoceros and Swamp deer accounting for dependency of both grazers on common food resource has been considered. The fixed point for three dimensional nonlinear systems at which all the species remain conserved with the stability conditions have been estimated. Using the parametric values given in census report of Kaziranga National park the future population of Indian Rhinoceros, Bengal Tiger and Swamp deer have been predicted and numerically verified with phase space and time series plots. Using simulation analysis with base value for 1999, the future value for all the three species have been estimated for the year 2016. It is observed that the forecasted values for Bengal tiger are closest to the actual value of population and the ecosystem remains in coexistence phase despite of poaching and lower growth rate of Bengal Tiger.

Keywords: nonlinear interactions, predator-prey model, stability analysis, coexistence, ecological system.

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1. Introduction

In nature, ecosystem is a functional unit where interactions between different species and the surrounding environment occur. In an ecosystem, evolution of the dynamics is governed by the various species reaction. In an ecology, every species interaction has direct and indirect dependence on each other's population growth with its own specific coefficient. The nonlinear mathematical model of inter-species interaction in the ecosystem helps to understand the evolution of population trajectories in phase space providing a projection of the possible states of stability or instability of the system of study.

Conservation of biological diversity has become a serious issue in recent times and requires immediate attention as in nature the ecological cycles are getting disturbed. Every species plays a crucial role in the food chain and if one single species becomes extinct the whole food chain gets affected which in turn disturbs the ecological cycles in nature. To save these species and to maintain the biodiversity, national parks, wildlife sanctuaries and bio-reserves has been developed across the globe. In 1972, the Indian Parliament enacted the first wild life protection act for the protection of plant and animal species with development of national parks in early 20th century A.D.

Kaziranga National park in India is one of the oldest national park developed in 1968 for the conservation of one horned Indian Rhinoceros. It is located in Assam, India 26o 40'N 93o21'E with an area of 430 square kilometre and was set up under Assam National Park Act of 1968. In this sanctuary, almost two third of world's great one horned rhinoceros are conserved and in 2006, it was declared as Tiger reserve. In Kaziranga National park Sambar Deer, Hog deer, Asian Elephant, Wild Buffalo and Wild Boar are the other major species which along with Bengal Tiger, Indian Rhinoceros and Swamp Deer constitute the overall fauna of this ecosystem.

Volterra [18] mathematically studied the mutual coexistence of species in an ecosystem with fluctuations in species abundance. Elton and Nicholas [5] derived the data based analysis of biological species, studied the Canadian Lynx population based on records of Lynx Canadensis fur collections in Canada, discussed the periodicity of fluctuations of the Canadian Lynx population with an average cycle of 9.6 years which shows dependency on Snowshoe Rabbit population cycle. Vézina and Platt [17] using inverse method analysed the food web dynamics of under sampled oceanic environment for two areas on English coast among the material fluxes in surface water coastal system and observed common patterns during late summers which indicate that how directly or through micro grazer pathway the particulate primary production flows to mesozooplankton which in turn fuel the downward particulate flux for future generation. Freedman and Ruan [7] studied the exhibition of group defence for food chain in three species prey population and taking carrying capacity as bifurcation parameter with delay in the model Hopf bifurcation was observed. Limit cycles and chaos was observed by Caswell and Neubert [2] while studying the effects of nonlinear closure terms on system dynamics of plankton food chain. It was observed that the limit cycles and chaos appearing in the dynamics of the system didn't eliminated and the food chain dynamics didn't stabilize with increase in capita mortality rate regardless of increase in density dependent or independent component.

Chauvet et al.[3] modelled the three species food chain of prey, predator and super-predator system using simulations and fixed point analysis the predator is observed to act as a conduit between prey and super-predator with possibility of extinction of predator and persistence of prey. Hsu et al. [8] discussed three and two trophic level food chain model and derived ratio dependent Michaeles-Menten type functional response conditions for extinction of three species in the form of stable steady state and limit cycle. The model had two distinct and realistic features of ratio dependence which produced extinction of prey species leading to collapse of system and dependence of system dynamics on initial population level showing feasibility of biological control in such systems. The predator-prey food chain between pest and its natural enemy in food chain with Holling type-III interaction was studied by Su et al. [12]. The pest eradication periodic solution was observed to be globally asymptotically stable when impulsive period is less than a threshold. Further, when the system is permanent it exhibits complex dynamic behaviour for the numerical simulation of impulsive perturbations on system dynamics. In study of three trophic aquatic food chain by Upadhyay [14], a new three species model based on Leslie-Gower scheme with incorporation of mutual interference in all three species is proposed. The mutual interference acts as a stabilizing and destabilizing factor for chaos. The toxic substance released by TPP population was act as a biological control which bring system to order from chaos. Ji et al.[9] discussed predator-prey model with Leslie Gower and Holling type –III schemes and stochastic perturbations. Through numerical simulation a positive solution exist with positive initial values for the long term behaviour of the system and conditions of extinction and persistence evaluated.

Etua and Rösseau [6] observed three kinds of regimes for extinction, coexistence and oscillatory phase of system dynamics in prey harvesting phenomenon with Gauss model and Holling type-III functional response for accounting interactions in the model. Application of such model in construction of real food chains for bio conservation and pest management has been highlighted. Evidence of reduction in resilience of age structured population by anthropogenic stressors for Daphnia population exposed to environmental stress was provided by Ottermanns et al. [10]. The chaos in population dynamics is observed to get repressed by enhancement of disturbance which increases the degree of synchrony. It is concluded that exposure to high concentration of chemical stressors can lead to increase in risk of extinction of Daphnia population. The above mentioned works clearly show that parameters of the ecosystem govern the direction in which system evolves. These parameters can be controlled in conservation process of bio-reserves to prevent extinction of any species for survival of other species and thus promoting coexistence of all.

Updhyay et al. [15,16] studied the emergence of spatio temporal patterns in their mathematical model for wetland Keoladeo National Park while Sahoo et al. [11] carried out their study on diseased prey model with Holling type interaction function. In this paper, non-linear interactions between the Indian Rhinoceros, Bengal Tiger and Swamp Deer for the Kaziranga ecosystem have been modeled. The species interaction type are Lotka-Voltera type and logistic growth has been considered for each species [15]. The competition between two prey species for common food resource with indirect dependency has been discussed with the mutual interaction of both the prey species with predator in the confinements of the protected environment of the national park. It is assumed that in the national park the amount of food resource is uniformly available in adequate abundance for both the prey species. With passage of time species population grow with birth of new prodigy, leading to load generation on the resource as consumption becomes higher which leads to increase in encounters between the members of species. Rhinoceros and Swamp deer compete occasionally thus both have an indirect dependency on each other's growth. As both the species are grazers and grass is the common food resource which they share. Thus growth of one species population casts a negative effect on the growth of other as rise in population leads to resource crunch for other leading to an indirect dependency. The factor of poaching has been accounted for the tiger and Rhino species national park based on the previous studies[1,4].

Using stability analysis, phase plot simulation, forecasting and validation for the system the current population level of all the three species are estimated which are observed to be mutually sustaining in the ecosystem of the Kaziranga National Park. All the three species show growth and tend to coexist mutually as per the simulation and population estimates of 2016 of Kaziranga National Park. To the best of author's knowledge, none of the authors have studied the nonlinear dynamics of Kaziranga ecosystem considering species interactions between the Indian Rhinoceros, Bengal Tiger and Swamp Deer.

2. Mathematical model

For modeling the Kaziranga national park the intermediate model is considered where one prey species acts as an intermediate between the other prey and predator species. Bengal Tiger(x) and Swamp Deer(y) are considered as predator and prey species while Indian Rhinoceros (z) is the intermediate species. The growth rate of Bengal Tiger, Swamp Deer and Indian Rhinoceros is represented by parameter a, e and i respectively. The value of a, e and i are considered to be positive for the protective ecosystem of Kaziranga where the growth of these species is encouraged for their conservation. As prey Bengal Tiger prefer hunting big ungulates like Chital, Sambar and Gaur. To a lesser extent they hunt Swamp Deer, Water Buffalo, Nilgai, Serow and Takin. As per census of 1999, the population of Sambar Deer was very low compared to that of Swamp Deer. Thus in this model of Kaziranga ecosystem Swamp deer is considered the species on which Tiger feeds with their interaction coefficient given by parameter $b(>0)$. Tigers rarely hunt Rhinoceros but in Kaziranga and Dudhwa National parks the killings of Rhinoceros by Tigers was reported. To account this fact, the interactions between Rhinoceros and Tigers are incorporated in the model and the interaction coefficient is represented by parameter $f(>0)$. Each species is considered to follow logistic growth model as the national park has a particular carrying capacity for each species.

The interaction of Rhinoceros and Tiger is considered to affect growth rate of Tiger negatively as Rhinoceros is a large hunt for the tiger to prey on alone which leads to rise in infighting between the tigers for dominance and larger share of food. Further due to horn Rhinoceros can kill or wound the Tiger which may lead to death. During the infighting on share of food and dominance tigers can get hurt badly or chased away. In both cases due to starvation or septicemia the death can be caused to the Tiger. The chased off tigers can approach towards the borders of national park where they can be targeted easily either by poachers or by human residing there. The interaction of Tiger and Rhinoceros positively impacts the swamp deer population. Being a large hunt Rhinoceros is capable of filling the appetite of the Tiger sufficiently leaving it with no requirement for hunting the Swamp Deer frequently. Thus a hunted Rhinoceros saves many swamp deer getting hunted. The Swamp Deer similarly contributes to the Rhinoceros population by being a smaller and readily available hunt to the tigers in large numbers in groups. As both Rhinoceros and Swamp Deer are grazers by virtue of their food habits they cast indirect negative impact on each other's population because as one grown in number it grazes more to cause less availability of fodder for the other to feed upon and thus bring its growth down.

The carrying capacity of the Kaziranga ecosystem for Bengal tiger, Swamp Deer and Indian Rhino is given by $k(>0)$, $l(>0)$ and $q(>0)$ respectively. The incidents of poaching of Bengal Tiger and Indian Rhinoceros has been reported in the Kaziranga ecosystem and till date poaching is taking place despite of measures being taken to curb it. The poaching of Bengal Tiger and Indian Rhinoceros has been taken into account in the model with $c(>0)$ and $g(>0)$ representing the poaching coefficients or number of incidents of poaching per year of both the species respectively. The three dimensional non-linear model can be written as following system of equations:

$$\dot{x} = f(x,y,z) = x(a - dx + by - fz - c), \tag{1}$$

$$\dot{y} = g(x,y,z) = y(e - hy - bx) + fxz, \tag{2}$$

$$\dot{z} = h(x,y,z) = z(i - jz - g - fx) + bxy, \tag{3}$$

where $d = \frac{a}{k}$=coefficient of intra-species interaction of Bengal Tiger population, $h = \frac{e}{l}$=coefficient of of intra-species interaction of Swamp Deer population, $j = \frac{i}{q}$=coefficient of of intra-species interaction of Indian Rhinoceros population.

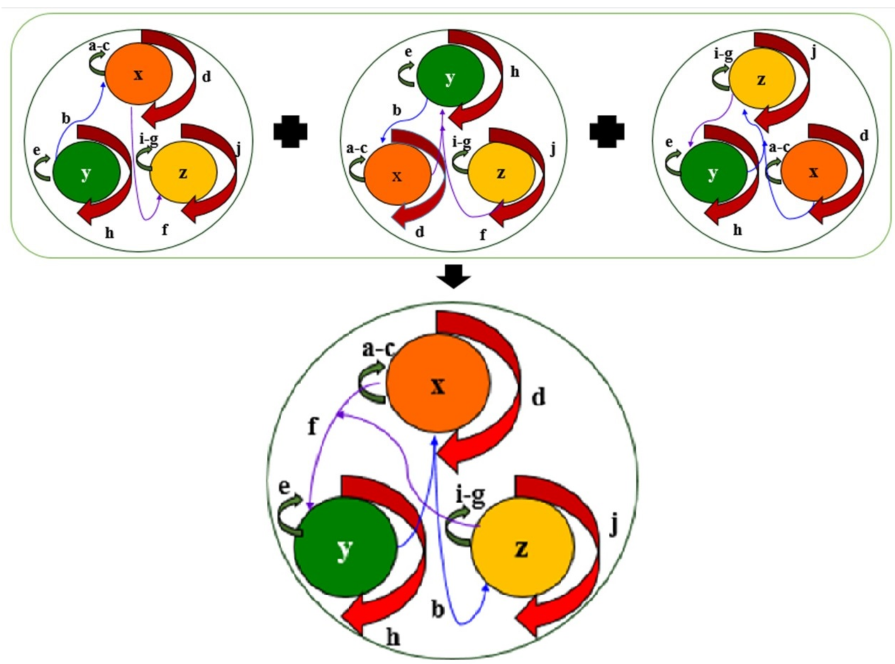

Figure 1: Schematic diagram of the Ecological Model $eq^n 1) - 3)$

3. Stability analysis

For the considered system fixed points are obtained equating $eq^n(1)-(3)$ simultaneously to zero i.e. $f(x,y,z) = g(x,y,z) = h(x,y,z) = 0$. The fixed points are mentioned as follows:

$$\begin{aligned}
1) \quad (x,y,z) &= (0,0,0) \\
2) \quad (x,y,z) &= \left(0, \frac{e}{h}, \frac{(i-g)}{j}\right) \\
3) \quad (x,y,z) &= \left(\frac{1}{d}\left[a + \frac{be}{h} - \frac{f(i-g)}{j} - c\right], \frac{e}{h}, \frac{(i-g)}{j}\right)
\end{aligned}$$

Jacobian of the system is as follows:

$$J = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial z} \\ \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} & \frac{\partial h}{\partial z} \end{bmatrix} = \begin{bmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{bmatrix}$$

where $m_1 = -2dx + a + by - fz - c$,$m_2 = bx$,$m_3 = -fx$,$m_4 = fz - by$,$m_5 = e - 2hy - bx$,$m_6 = fx$,$m_7 = by - fz$,$m_8 = bx$ and $m_9 = i - 2jz - fx - g$. The characteristic polynomial is expressed as follows:

$$\begin{vmatrix} m_1 - \lambda & m_2 & m_3 \\ m_4 & m_5 - \lambda & m_6 \\ m_7 & m_8 & m_9 - \lambda \end{vmatrix} = \lambda^3 + e_1\lambda^2 + e_2\lambda + e_3 = 0$$

where $e_1 = -(m_1+m_5+m_9)$, $e_2 = m_1(m_5+m_9)+m_5m_9-m_6m_8-m_2m_4-m_3m_7$ and $e_3 = -m_1m_5m_9-m_2m_6m_7-m_3m_4m_8+m_6m_8m_1+m_2m_4m_9+m_3m_7m_5$. For a characteristic polynomial $\lambda^3+e_1\lambda^2+e_2\lambda+e_3 = 0$ as obtained from Jacobian at a particular fixed point, if roots have negative real parts with $e_1 > 0$, $e_3 > 0$ and $e_1e_2 - e_3 > 0$ then this implies stability for that fixed point by Routh-Hurwitz Criteria(RHC).

Case I: For the first fixed point $FP_1= (x, y, z) = (0, 0, 0)$, we obtain $m_2 = m_3 = m_4 = m_6 = m_7 = m_8 = 0$ and $m_1 = a - c$,$m_5 = e$, $m_9 = i - g$. Thus we get a characteristic polynomial $\lambda^3 + e_1\lambda^2 + e_2\lambda + e_3 = 0$, where $e_1 = (c + g) - (a + e + i)$;$e_2 = e(a - c) + (e + a + -c)(i - g)$;$e_3 = e(c - a)(i - g)$. It is observed that for stability of the fixed point FP_1 following conditions are required to be satisfied:

- For $e_1 > 0$, $a + e + i < c + g \to s_1 < s_2$ (condition 1).
- For $e_3 > 0$, $c > a$ and $i > g$ (condition 2).
- For $e_1e_2 - e_3 > 0$, $4(c-a)(g-i) < e^2(c-a+g-i)+(c-a)^2(g-i-e)+(g-i)^2(c-a-e) \to s_3 < s_4$ and $c > (a + e)$;$g > (i + e)$ (condition 3).

Case II: For the second fixed point $FP_2= (x, y, z) = \left(0, \frac{e}{h}, \frac{(i-g)}{j}\right)$, we obtain $m_2 = m_3 = m_6 = m_8 = 0$ and $m_1 = a+\frac{be}{h}-\frac{f(i-g)}{j}-c$,$m_4 = \frac{be}{h}-\frac{f(i-g)}{h}$,$m_5 = -e$, $m_9 = -i + g$. Thus we get a characteristic polynomial $\lambda^3 + e_1\lambda^2 + e_2\lambda + e_3 = 0$ where $e_1 = -\left(a + \frac{be}{h} - \frac{f(i-g)}{j} - c - e + g - i\right)$;$e_2 = \left(a + \frac{be}{h} - \frac{f(i-g)}{j} - c\right)(g - e - i) + f(i - g)$;$e_3 = e\left(a + \frac{be}{h} - \frac{f(i-g)}{j} - c\right)(g - j)$. It is observed that for stability of the fixed point FP_2 following conditions are required to be satisfied:

- For $e_1 > 0$, $i > g$;$\frac{f(i-g)}{j} > \frac{be}{h} \to s_5 > s_6$ and $(c + e) + (i - g) > \left(a + \frac{be}{h} - \frac{f(i-g)}{j} - c\right) \to s_7 > s_8$(condition 4)
- For $e_3 > 0$, $g < i$ and $a + \frac{be}{h} < \frac{f(i-g)}{j} + c \to s_9 < s_{10}$ (condition 5)
- For $e_1e_2 - e_3 > 0$, $\left((c + e) + (i - g) - a - \frac{be}{h} + \frac{f(i-g)}{j}\right)\left(\left(a + \frac{be}{h} - \frac{f(i-g)}{j} - c\right)U_1 + f(i - g)\right) < e(i - g)\left(\frac{f(i-g)}{j} + c - a - \frac{be}{h}\right) \to s_{11} < s_{12}$, where $U_1 = (g - e - i)$ (condition 6)

Case III: For fixed point $FP_3 = (x, y, z) = \left(\frac{1}{d}\left(a + \frac{be}{h} - \frac{f(i-g)}{j} - c\right), \frac{e}{h}, \frac{(i-g)}{j}\right)$, we obtain $m_1 = \left(\frac{f(i-g)}{j} + c - a - \frac{be}{h}\right)$, $m_2 = -m_1\frac{b}{d}$,$m_3 = m_1\frac{f}{d}$,$m_4 = m_1 - c + a$,$m_5 = -e - m_2$,$m_6 = -m_3$,$m_7 = -m_4$,$m_8 = m_2$,$m_9 = g - i + m_3$. Thus we get a characteristic polynomial:

$$\lambda^3 + e_1\lambda^2 + e_2\lambda + e_3 = 0,$$

where $e_1 = -\left(m_1 + \left(-e + \frac{bm_1}{d}\right) + (g-i) + \frac{fm_1}{d}\right)$, $e_2 = m_1m_5 + m_1m_9 + m_5m_9 + m_3m_2 + (m_3 - m_2)m_4$ and $e_3 = -m_1m_5m_4 - m_2m_4(i-g) - m_3m_2m_1 + m_3m_4e$. It is observed that for stability of the fixed point FP_3 following conditions are required to be satisfied:

- For $e_1 > 0$, $i > g$ and $(i + e - g) > m_1\left(1 + \frac{(b+f)}{d}\right) \to s_{13} > s_{14}$ (condition 7)
- For $e_3 > 0$, $g < i$ and $(2m_1 + (a-c))(m_3e - m_2(i-g)) > em_1(i-g) \to s_{15} > s_{16}$ (condition 8)
- For $e_1e_2 - e_3 > 0$, $\left(i + e - g - m_1\left(1 + \frac{(b+f)}{d}\right)\right)(m_1m_5 + m_9(m_1 + m_5) + m_3m_2 + (m_3 - m_2)m_4) > ((2m_1 + a - c)(m_3e - m_2(i-g)) - em_1(i-g)) \to s_{17} > s_{18}$ (condition 9)

4. Results and discussion

As per 1999 census report for Kaziranga National Park, the average number of instants of Tigers attacking Rhinoceros were twelve from 1985-2000 [13]. Thus $f = 12$ and it is assumed that $b = 36$ as Swamp Deer with height and weight being almost half and one tenth of height and weight of a Rhinoceros respectively. Also, in 1999 the population of Bengal tigers, Indian Rhinoceros and Swamp Deer were 80, 398 and 1552 respectively which has increased to 111, 1148 and 2401 respectively as per population estimation of Kaziranga National Park for year 2016. Thus for simulation and validation, the initial population taken as $x0 = 80$, $y0 = 398$ and $z0 = 1552$. The growth rates a, e and i from 1999 to 2016 taken as $a = 1.44$, $e = 46.875$ and $i = 53.06$. As per the report submitted by Kaziranga Field Director to Guwahati High Court, the carrying capacity for rhinoceros, tiger and swamp deer are 2750, 146 and 800 respectively. Thus $d = 0.009$, $h = 0.058$ and $j = 0.01953$ respectively. Table 1, Table 2 and Table 3 gives the details about different conditions 1-9 obtained for three points and the behaviour of each point is discussed. For $FP_1 = (0,0,0)$ only Condition 2

Table 1: Stability of fixed point FP_1 at a =1.44, b =36, c =22, d =0.009, e =46.875, f =12, g =11, h =0.058, i =53.06, j =0.01953

Condition No.	Observation	Status
Condition 1	$s_1 > s_2$	Not Satisfied
Condition 2	$c > a$ & $i > g$	Satisfied
Condition 3	$c < a + e$, $g < i + e$ & $s_3 > s_4$	Not Satisfied

is satisfied and thus it is not stable for the considered system parameter values. For $FP_2 = (0, 808, 2153)$ none of the conditions are satisfied and thus it is not

Table 2: Stability of fixed point FP_2 at a =1.44, b =36, c =22, d =0.009, e =46.875, f =12, g =11, h =0.058, i =53.06, j =0.01953

Condition No.	Observation	Status
Condition 4	$s_5 < s_6$ & $s_7 < s_8$	Not Satisfied
Condition 5	$g < i$ & $s_9 > s_{10}$	Not Satisfied
Condition 6	$s_{11} > s_{12}$	Not Satisfied

Table 3: Stability of fixed point FP_3 at a =1.44, b =36, c =22, d =0.009, e =46.875, f =12, g =11, h =0.058, i =53.06, j =0.01953

Condition No.	Observation	Status
Condition 7	$g < i$ & $s_{13} > s_{14}$	Satisfied
Condition 8	$s_{15} > s_{16}$	Satisfied
Condition 9	$s_{17} > s_{18}$	Satisfied

stable for the considered system parameter values. For $FP_3 = (126, 808, 2153)$ all of the conditions are satisfied and thus it is stable for the considered system parameter values. Thus, from stability analysis the predicted values for tiger, rhinoceros and swamp deer population are 126, 808 and 2153 respectively for the year 2016.

From the study, it is evident that the ecosystem in Kaziranga is in stable co-existence phase and will remain in it till these parameters values are maintained. The projected trajectory that species population takes in phase space simulation is shown in Figure 2 a) which verify the phase of coexistence in Kaziranga ecosystem. Using simulation, for the year 2016 the predicted values for tiger, rhinoceros and swamp deer population are 133, 741 and 2220 respectively. The stability of fixed point is verified using Lyapunov exponent which are plotted in Figure 2 b). As the values of Lyapunov exponents are negative ($\lambda_1 = -15.888$, $\lambda_2 = -24.262$, $\lambda_3 = -43.368$) thus, system shows stable behavior. Predicted and observed values for population of Bengal Tiger, Indian Rhinoceros and Swamp Deer for the year 2016 are shown and compared in Table-4.

From Table 4, it is evident that the population tend to increase for all the three species in Kaziranga ecosystem as predicted. For Bengal Tiger and Indian Rhinoceros, the predicted values are close to the actual values but for Swamp Deer the predicted values are deviated from the actual value. The deviation between observed and predicted value exists as continuous species population data is unavailable to determine the exact parameters. However, it is observed that the predicted values for Bengal Tiger and Indian Rhinoceros are in sync with the observed values which indicates that their interactions in the ecosystem has been properly modeled. Despite the poaching phenomena the projected

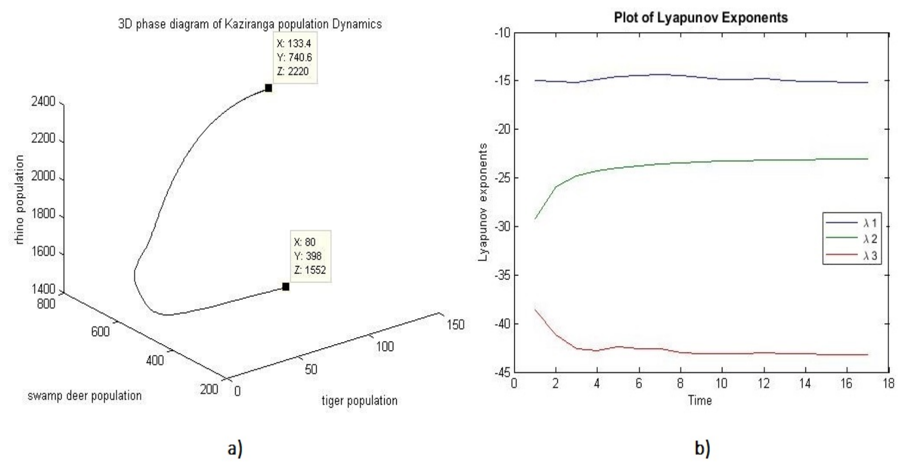

Figure 2: a) Phase Space Plot b) Plot of Lyapunov Exponents for system $eq^n 1)-3)$ at a =1.44, b =36, c =22, d =0.009, e =46.875, f =12, g =11, h =0.058, i =53.06, j =0.01953

Table 4: Predicted (P) and Actual (A) values for population of Bengal Tiger, Indian Rhinoceros and Swamp Deer using fixed point (FP) and phase plot (PP) analysis

Species	2016(P-FP)	2016(P-PP)	2016(A)
Bengal Tiger	126	133	111
Indian Rhino	2153	2200	2401
Swamp Deer	808	741	1148

trajectory of the system model does exhibit a significant rise in Rhino population as observed in previous studies[1,4]

5. Conclusion

In this paper the nonlinear interactions between tiger, rhinoceros and swamp deer in the ecosystem of the Kaziranga national park has been modeled with basic Lotka-Volterra type species interaction function. The species population was predicted using simulation for year 2016 based on the population data obtained from 1999 census report of different species in Kaziranga National Park. The predicted values of the population show an increasing trend as observed in the actual estimation of population for 2016. The predicted values of population deviate from actual values but the interaction dynamics between the rhino and tiger population are well simulated and model nearly establishes the basic species interaction dynamics which is evident from the predicted values which are observed to be in close range of the actual values. From the stability analysis, the conditions for stable coexistence are determined. It is concluded that if the values of system parameters are maintained without much variation then despite of poaching the population of all the species can be sustained mutually.

Acknowledgement

Authors are thankful to Guru Gobind Singh Indraprastha University (GGSIPU), Delhi(India) for providing research facilities. Saureesh Das would like to thank GGSIPU for financial support.

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Accepted: February 02, 2021