A mathematical model of white rhino translocation strategy

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Zahra Zahira and Dipo Aldila

A Mathematical Model of White Rhino Translocation Strategy

Zahra Zahira and Dipo Aldila[a)]

Department of Mathematics, Universitas Indonesia, Kampus UI Depok, Depok 16424, Indonesia.

[a)] *Corresponding author: aldiladipo@sci.ui.ac.id*

Abstract. Southern white rhinoceros (*Ceratotherium simum simum*) is a species of rhino that is categorized as near threatened in the world. Most population of white rhino can be found in South Africa. According to International Union for Conservation of Nature (IUCN), a strategy is needed to conserve and increase the white rhino population. Hence, a mathematical model of the white rhino translocation strategy will be introduced. The model is constructed by dividing the white rhino population into four subpopulations based on their habitat and maturity. From the mathematical analysis, we obtain three types of equilibrium points. Furthermore, we investigate the existence and local stability criteria of these equilibrium points analytically and numerically. It is shown that, when translocation rate implemented in a proper way, translocation can accelerate the growth rate of the white rhino population both in the source or translocation habitat.

INTRODUCTION

The white rhino (*Ceratotherium simum*) is one of five rhino species that still survive on the African continent. According to the International Union for Conservation of Nature (IUCN) Red List, the southern white rhino is now classified as "Near Threatened" with approximately 10,080 adult southern white rhinos [1]. Due to the increasing threat of loss of this species, much effort has been made to combat white rhino poaching and address its impacts. One of them, research conducted in Kruger National Park which shows that the birth rate can decrease with increasing density and decreasing rainfall [2]. Therefore, strategically relocating rhinos is considered as one of the relevant ways to protect and control the white rhino population. So, to increase the white rhino population, one of the conservation strategies is translocation.

Translocation has become routine in many African states and has played a big role in improving the number of white and black rhinos [2]. Translocations in any animal species, especially rhinos, are very taxing on the animal and this can affect the success of translocation attempts. Translocation involves capture, temporary captivity, transport, and releasing white rhinos to a new environment, exposing the animal to a variety of stressors such as prolonged periods of water and food deprivation that can cause morbidity or mortality [3]. Therefore, stress should be considered as a factor that should be integrated into the translocation strategy [4]. The biggest stress factor for rhinos when they are transferred is the big change from their natural environment to the new environment they are translocated to [5]. Rhinos often traumatize themselves during transport [6] or become sick and die after being released [2]. The current mortality rate for rhino translocations in South Africa and Namibia is estimated at 5 percent [7].

Once translocated, rhinos change their behavior as they become more familiar with their new environment. In addition, rhinos that were translocated earlier also directly or indirectly influenced the establishment of home ranges and the behavior of the later released rhinos. Newly translocated rhinos usually avoid areas that have been occupied by previously translocated rhino groups. Fighting is also very common among adult male rhinos. In these fights, the newcomer rhinos always lose and then moved away from territories that are occupied [8].

To the best of our knowledge, not so many mathematical models consider the white rhino population dynamic, including their translocation strategy. Rhinoceros, both black and white rhinos use translocation as one of the conservation efforts. However, the population of black rhinos is much less than the population of white rhinos. Behaviorally, black rhinos have a reputation for being more aggressive and territorial than white rhinos [2]. This may be the reason why black rhinos although translocation conservation efforts have been carried out, the population cannot increase significantly like the white rhinos. In 2012, Aldila, et al. [9] discuss the translocation strategy of black rhino in their proposed model. They found that the higher carrying capacity of the targeted translocation area will increase the chance that translocation strategy success. In 2020, Aldila et al. [10] proposed an optimal control model to understand the impact of illegal poaching on the rhino population. They found that the price of rhino horn on the market should be controlled carefully to suppress the number of hunters in the field.

Therefore, based on the explanation above, this paper aims to construct a model of the white rhino translocation strategy which considers two ecosystems, internal competition between rhino, and translocation. Existence and stability criteria of extinction and persistence of equilibrium points were analyzed in detail. Some numerical simulations were conducted to see the impact of the change in translocation strategy on the dynamics of white rhino.

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MATHEMATICAL MODEL

A translocation model for white rhino presented in this section. We begin by define our variables, namely the rhino population from source area (S), adult rhino who just recently translocated and not yet adapt to the new environment (R), adapted rhino (A), and juvenile rhino (C). Please note that R,A and C are stay in a same ecosystem, namely translocation area, while S is in the source area.

We assume S is growth with a logistic rate with a constant rate r and carrying capacity K_1. Translocation strategy conducted by transfer some of population of S with a rate of u into translocation area. Hence, R population increase by uS. In addition, we assume that rhino who just recently translocated (R), can not adapt quickly, and could be death during contact to rhino who already stay in the area. We assume that this contact could end up with death of R with a contact coefficient β, and also could death during adaptation due to natural reason with a rate of ε. If T succeed to adapt, they will transfer to adapted rhino population, with a rate of γ. We assume that only A population could conduct a reproduction. Hence, C increase with a logistic rate of r and carrying capacity of K_2. We assume that C will be transferred into A with a rate of α due to age. We assume that all compartments conduct death rate of μ. The detailed flow chart of our model could be seen in Fig. .

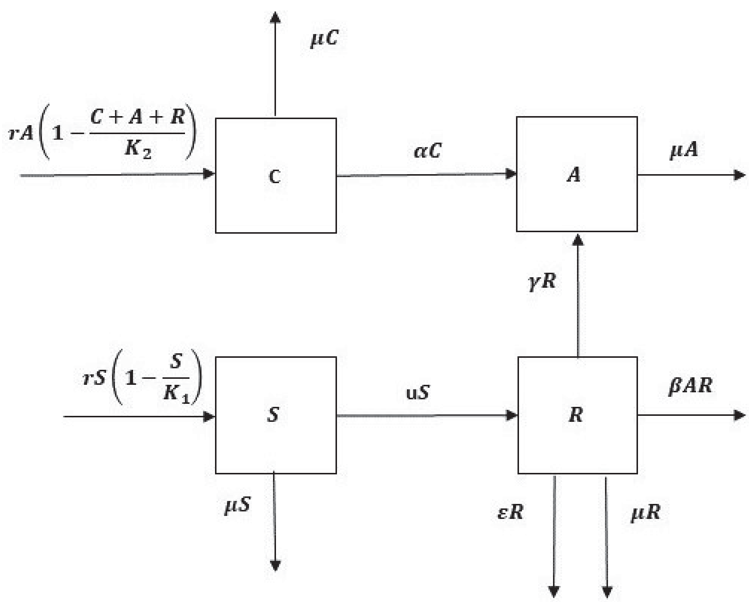

FIGURE 1. Flowchart diagram for population dynamic model in (1)

Using the assumptions stated above and Figure 1, the system of differential equations for the white rhino translocation strategy is given by

$$
\begin{aligned}
\frac{dS}{dt} &= rS\left(1-\frac{S}{K_1}\right) - uS - \mu S,\\
\frac{dR}{dt} &= uS - \varepsilon R - \mu R - \beta AR - \gamma R,\\
\frac{dC}{dt} &= rA\left(1-\frac{C+A+R}{K_2}\right) - \alpha C - \mu C,\\
\frac{dA}{dt} &= \alpha C + \gamma R - \mu A,
\end{aligned}
\tag{1}
$$

with positive initial conditions. We assume that all parameters are positive, and summed up in Table I.

TABLE I. Description of parameters in system (1).

Par.	Description	Value	Unit
r	Birth rate of the white rhino	$r \in (0,\infty)$	year^{-1}
K_1, K_2	Carrying capacity for source and juvenile white rhino population	$K_1, K_2 \in (0,\infty)$	rhino
α	Transition rate from juvenile rhino to adult rhino	$\alpha \in (0,\infty)$	year^{-1}
μ	Rate of white rhino's natural death	$\mu \in (0,\infty)$	$\frac{1}{\text{time}}$
ε	Rate of white rhino's death due to stress	$\varepsilon \in (0,\infty)$	year^{-1}
u	Translocation rate of white rhino	$u \in (0,\infty)$	year^{-1}
γ	Transition rate or average time of successful R adaptation in a new habitat	$\gamma \in (0,\infty)$	year^{-1}
β	Rate of newcomer white rhino's death due to contact with adult rhino	$\beta \in [0,\infty)$	$(\text{rhino} \times \text{year})^{-1}$

MODEL ANALYSIS

In this section, we will analyze the existence and local stability of all equilibrium points of the system (1). First, the equilibrium points are done by taking the right-hand side of the system (1) equal to zero and solve the system (1) respect to all variables. We get three equilibrium points as follows:

1. The extinction equilibrium, i.e. $\Omega_0 = (0,0,0,0)$. This equilibrium point represents the situation where all populations are extinct.

2. The source and newcomer rhino free equilibrium given by
$$\Omega_1 = (0,0,C^*,A^*) = \left(0,0,\frac{\mu K_2(r-\mu)(1-\mathscr{R}_2)}{r(\alpha+\mu)},\frac{K_2[\alpha(r-\mu)](1-\mathscr{R}_2)}{r(\alpha+\mu)}\right)$$
where $\mathscr{R}_2 = \frac{\mu^2}{\alpha(r-\mu)}$. It is easy to see that Ω_1 has a biological interpretation (exist) if (a) $r-\mu < 0$ or, (b) $r-\mu > 0$ and $\mathscr{R}_2 > 1$. This equilibirum present a condition when only the translocated population (juvenile and adult white rhino) exists.

3. The coexistence equilibirum given by $\Omega_2 = (S^*,R^*,C^*,A^*)$. This equilibrium appears in implicit form as a function of A as follows
$$\begin{aligned} S^* &= -\frac{A(rA(\alpha+\mu)-K_2((-\mu+r)\alpha-\mu^2))(\beta A+\varepsilon+\gamma+\mu)}{u(-\gamma K_2(\alpha+\mu)+rA(\alpha-\gamma))}, \\ R^* &= \frac{A(K_2((-r+\mu)\alpha+\mu^2)+rA(\alpha+\mu))}{\gamma K_2(\alpha+\mu)-rA(\alpha-\gamma)}, \\ C^* &= \frac{rA(\gamma(A-K_2)+\mu A)}{(-rA-K_2\gamma(\alpha+\mu))+r\alpha A}, \end{aligned} \tag{2}$$
and A^* is a solution of a three-degree polynomial equation in the form of
$$F(A) = a_3A^3 + a_2A^2 + a_1A + a_0 = 0$$
where $a_3 = \beta r^2(\alpha+\mu)$, $a_2 = r(\mu^2+(\alpha+\gamma+\varepsilon)\mu-\alpha(\beta K_2-\varepsilon-\gamma))+\mu\beta K_2(\alpha+\mu)$, $a_1 = r(((-\mu+r)\alpha-\mu^2)(\varepsilon+\gamma+\mu)K_2 - K_1u(\alpha-\gamma)(r-u-\mu))$, and $a_0 = \gamma K_1K_2u(\alpha+\mu)(\mu-r+u)$. It is easy to see that if $a_0 < 0 \iff r > (\mu+u)$, then we will always at least one positive root of $F(A)$. We analyze this polynomial using Descartes rules of sign and the result given in Table II.

TABLE II. Descartes Rules of Sign of polynomial when $a_3 > 0$

Case	a_3	a_2	a_1	a_0	**Changes of Sign**	**Total Positive Root**
1	+	+	+	+	0	0
2	+	+	+	-	1	1
3	+	+	-	+	2	0 or 2
4	+	+	-	-	1	1
5	+	-	+	+	2	0 or 2
6	+	-	+	-	3	1 or 3
7	+	-	-	+	2	0 or 2
8	+	-	-	-	1	1

Next, we analyze the local stability criteria of Ω_0 and Ω_1 analytically, while Ω_2 will be conducted numerically due to it complexity. The Jacobian matrix of system (1) is given as follows:

$$\mathscr{J} = \begin{bmatrix} r\left(1-\frac{S}{K_1}\right)-\frac{rS}{K_1}-u-\mu & 0 & 0 & 0 \\ u & -\beta A-\varepsilon-\gamma-\mu & 0 & -\beta R \\ 0 & -\frac{rA}{K_2} & -\frac{rA}{K_2}-\alpha-\mu & r\left(1-\frac{C+A+R}{K_2}\right)-\frac{rA}{K_2} \\ 0 & \gamma & \alpha & -\mu \end{bmatrix}. \tag{3}$$

Substitute Ω_0 into $\mathscr{J}$, and calculate the eigenvalues, we have 4 eigenvalues. Two of them are $\lambda_1 = -\varepsilon-\gamma-\mu$ which always negative, $\lambda_2 = r-u-\mu$ which is negative if $\mathscr{R}_1 > 1$ and the other two taking from the root of polynomial $(\lambda^2+(\alpha+2\mu)\lambda-\alpha(r-\mu)+\mu^2)$. It is easy to see that the two eigenvalues from this polynomial will be negative if $\mathscr{R}_2 > 1$. Thus, Ω_0 is stable if $\min\{\mathscr{R}_1,\mathscr{R}_2\} > 1$. This result tells us that if Ω_1 exists, Ω_0 will become unstable and vice versa.

For stability of Ω_1, linearization system (2) near the Ω_1 gives the characteristic equation of $\mathscr{J}|_{\Omega_1}$ given by

$$(a_2\lambda^2+a_1\lambda+a_0)(b_2\lambda^2+b_1\lambda+b_0) = 0,$$

where $a_2 = 1, a_1 = \dfrac{K_2\beta[\alpha(r-\mu)](1-\mathscr{R}_2)}{r(\alpha+\mu)}+(\varepsilon+\gamma+2\mu-r+u), a_0 = \left(-\dfrac{K_2\beta[\alpha(r-\mu)](1-\mathscr{R}_2)}{r(\alpha+\mu)}-\varepsilon-\gamma-\mu\right)(r-\mu-u), b_2 = 1, b_1 = \dfrac{\alpha^2+(r+2\mu)\alpha+\mu^2}{\alpha+\mu}$ and $b_0 = \alpha(r-\mu)-\mu^2$. From direct calculation, it is known that Ω_1 will be stable if $\min\{M_0,M_1,M_2\} > 0$ where

$$\begin{aligned} M_0 &= \left(-\frac{K_2\beta[\alpha(r-\mu)](1-\mathscr{R}_2)}{r(\alpha+\mu)}-\varepsilon-\gamma-\mu\right)(r-\mu-u), \\ M_1 &= \frac{K_2\beta[\alpha(r-\mu)](1-\mathscr{R}_2)}{r(\alpha+\mu)}-(r-\varepsilon-\gamma-2\mu-u), \\ M_2 &= \frac{\alpha^2+(r+2\mu)\alpha+\mu^2}{\alpha+\mu}. \end{aligned}$$

Lastly, numerical experiment indicates that the coexistence equilibrium Ω_2 is stable if $\mathscr{R}_1 < 1$. A summary of the positiveness and local stability criteria for each equilibrium point is presented in Table III.

Since $\mathscr{R}_1 < 1 \iff u < r-\mu$, we can conclude that the key factor that can affect the value of $\mathscr{R}_1$ is the translocation rate (u) since r and μ in unchangeable. We found that the condition $\mathscr{R}_1 < 1$ can be met if the value of u is less than the difference between the birth rate of the white rhino (r) and the natural death rate of the white rhino (μ). In other words, by controlling the rate of translocation, the white rhino population can exist both in source and translocation ecosystem.

TABLE III. Positiveness and local stability criteria for each equilibrium point.

Equilibrium Point	Positive criteria	Local stability criteria
$\Omega_0 = (0,0,0,0)$	-	$\min\{\mathscr{R}_1, \mathscr{R}_2\} > 1$
$\Omega_1 = (0,0,C^*,A^*)$	$r-\mu < 0$ or $r-\mu > 0$, $\mathscr{R}_2 < 1$	$\min\{M_0, M_1, M_2\} > 0$
$\Omega_2 = (S^*,R^*,C^*,A^*)$	$\mathscr{R}_1 < 1$	Stable if $\mathscr{R}_1 < 1$ (from numerical experiments)

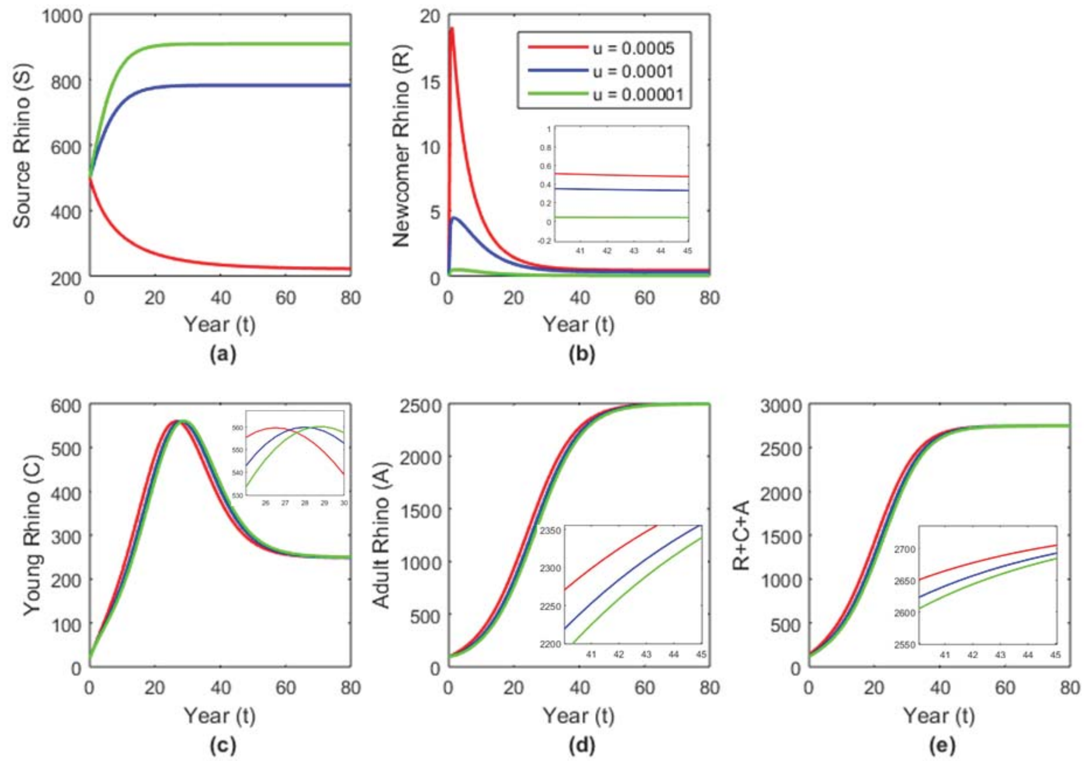

FIGURE 2. Dynamics of system (1) with variations of u to $t \in [0, 80]$.

NUMERICAL SIMULATION

In this section, numerical results are given in order to analyze the response of the white rhino populations to different translocation strategies.

Autonomous Simulation with Variation of Translocation Rate

In many cases, translocation attempts are not controlled. For example, in moving a population from a translocation source area to a translocation area, it is done without considering the sustainability of the source population. As a result, the number of displaced populations is so large that the initial population is unable to increase again.

In this section, autonomous simulation will be carried out to see how the translocation rate of white rhino (u) affects the dynamics of the model (1). The simulation is carried out with three different variations of the u value, namely $u = 0.0005$, $u = 0.0001$ and $u = 0.00001$. By substituting the values of other parameters as follows $r = \frac{13}{50\times365}, K_1 = 1000, K_2 = 3000, \alpha = \frac{1}{5\times365}, \mu = \frac{1}{50\times365}, \varepsilon = 0.00001, u = 0.00001, \gamma = \frac{1}{2\times365}, \beta = 0.0001$ and initial value $S(0) = 500, R(0) = 0, C(0) = 20, A(0) = 100$, the results are as shown in Figure 2. From the figure, it can be concluded that u really changes the equilibrium but is not very sensitive so that it does not change the equilibrium size drastically except for the source rhino population. However, u also affects the speed to reach a different equilibrium although it is not very sensitive. Therefore, translocation efforts must be controlled by maintaining the sustainability of the population in the initial area or source of the translocation area.

Autonomous Simulation with Variation of the Competition Coefficient

In this section, autonomous simulation will be carried out to see how the competition coefficient between the newcomer rhino and the adult local rhino (β) affects the dynamics of the model (1). The simulation is carried out with three different variations of the β value, namely $\beta = 0.0001$, $\beta = 0.000001$ and $\beta = 0.00000001$. By substituting the values of other parameters as follows $r = \frac{13}{50\times365}, K_1 = 1000, K_2 = 3000, \alpha = \frac{1}{5\times365}, \mu = \frac{1}{50\times365}, \varepsilon = 0.00001, u = 0.00001, \gamma = \frac{1}{2\times365}$ and initial value $S(0) = 500, R(0) = 0, C(0) = 20, A(0) = 100$, the result can be seen in Fig. 3. From the Fig. 3, it can be concluded that β really changes the equilibrium but is not too sensitive so that it does not change the equilibrium size drastically except for the newcomer rhino population. However, β also affects the speed to reach a different equilibrium although it is not very sensitive. Thus, the smaller the coefficient of competition between the newcomer rhino and adult local rhino, the newcomer rhino population will increase so eventually the white rhino population in the translocated habitat also increases.

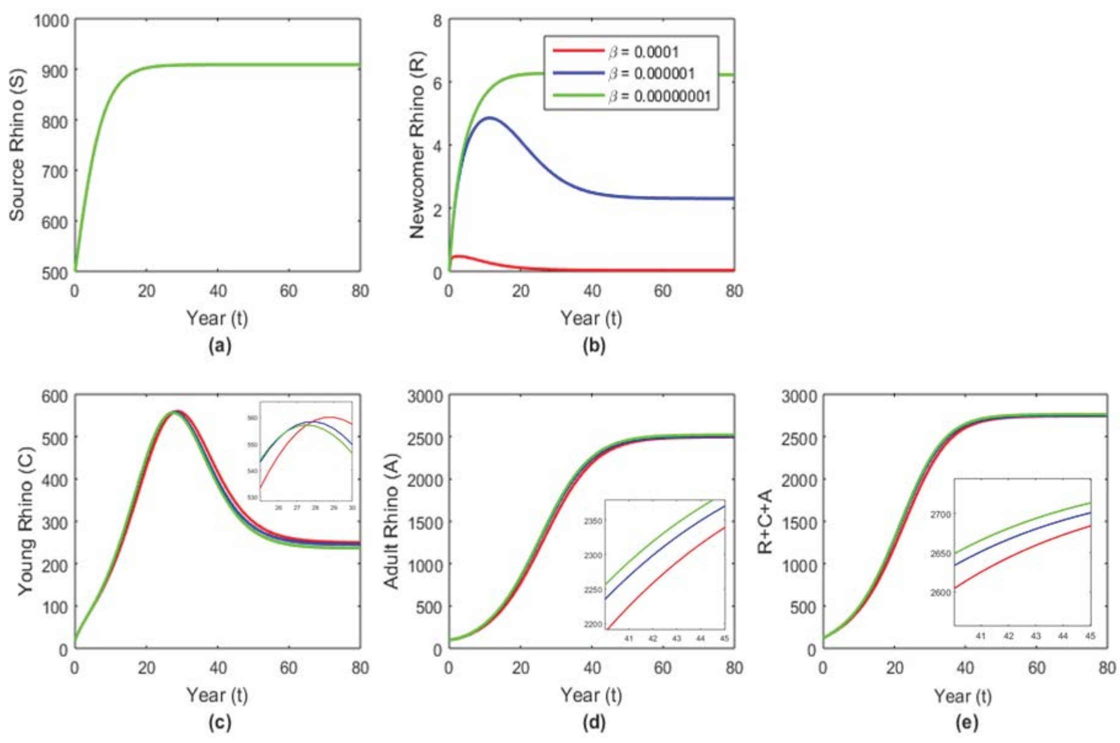

FIGURE 3. Dynamics with variation β for $t \in [0, 80]$

DISCUSSION AND CONCLUSION

In this paper, a mathematical model is constructed for the white rhino translocation strategy considering two ecosystems, namely source ecosystem, and translocation ecosystem. The existence and stability of the equilibrium were discussed analytically and numerically. We find that the uncontrolled translocation strategy can end up in the extinction of the rhino population in the sourcing ecosystem. In translocation, moving the white rhino population from one environment to a new environment requires monitoring. The translocation rate of white rhino needs to be monitored so that it is large enough to increase the population but not so large that it does not meet the requirements for coexistence between white rhino populations. In addition, the newly translocated rhino must be accustomed to adapting to the new environment first. Usually, there is a breeding ground in the new environment where the location must be large enough so that the white rhino can feel like living in the free world even though it is still in captivity. This aims to increase the adaptation rate of the newly translocated white rhino. At the same time, it is possible to reduce competition between newcomer rhinos and adult local rhinos by reducing the chances of the two rhino populations interacting (described by β in our model). This can be done by releasing newcomer rhinos into new areas that are far from the territory of adult local rhinos who already exist in the translocation area before.

In this article, we introduce a simple model to understand the impact of uncontrolled translocation strategy, and the impact of competition in the translocation area. However, our model has some lack on the factor that is important to consider. For further development of the model, the reader may consider the impact of external competition with other species in the translocation area, disease in the translocation area, and limitation of budget for translocation strategy. It is also important to see how the translocation strategy should be applied optimally. An optimal control problem from illegal poaching of white rhino reveals that intervention by the government must consider conditions in the field such as intervention costs, sale prices for rhino horns, and hunter populations [10]. Therefore, an optimal control problem of white rhino translocation strategy is also needed.

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REFERENCES

1. The IUCN Red List of Threatened Species 2020, *White Rhino*, https://www.iucnredlist.org/species/4185/45813880. (Accessed 20 Jan 2021)
2. R.H. Emslie, R. Amin, and R. Kock, Guidelines for the in Situ Re-Introduction And Translocation of African and Asian Rhinoceros. Gland, Switzerland: IUCN. (2009).
3. C.P. Teixeira, C.S. de Azevedo, M. Mendl, C.F. Cipreste, and R.J. Young, Anim. Behav. 73, 1 (2007).
4. M.J. Dickens, D.J. Delehanty, and L. Michael Romero, Biol. Conserv. 143, 1329 (2010).
5. J.W. Turner, P. Tolson, and N. Hamad, J. Zoo Wildl. Med. 33, 214 (2002).
6. A. Morkel, P.; Kennedy-Benson, B. by FZS, SAVE Found. Aust. Namibian Minist. Environment Tour. Save Rhino Trust p. 85. (2007).
7. M. Miller, M. Kruger, M. Kruger, F. Olea-Popelka, and P. Buss, J. Wildl. Dis. 52, S78 (2016).
8. O. Berger-tal and D. Saltz, Curr. Biol. 60, 515 (2014).
9. D. Aldila, A.J. Hutchinson, M. Woolway, , N. Owen-Smith, J. Math. Fund. Sci. 47, 104 (2015).
10. D. Aldila, N. Azizah, and B.D. Handari, Adv. Differ. Equations 2020, (2020).