Logistic Model as A Representation of *Rhinoceros sondaicus* and *Bos javanicus* Population at Ujung Kulon National Park

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Abstract. Ujung Kulon National Park is located in Banten Province, Indonesia. It is known as the last wildlife sanctuary for Javan rhino (Rhinoceros sondaicus) [7]. Recent report from the Ujung Kulon National Park indicates that the rhino population remains stagnant at the level of 50s for long period of time. In this article, the authors try to represent the state of the rhino population in Ujung Kulon National Park using logistic type equation associated with the presence of wild bull (Bos javanicus) populations. Here the model of carrying capacities are function of both species. Analysis of co-existence and its stability are done here. Numerical simulation is shown to provide an overview of the interactions that occur between the two species above. The results obtained from this modeling conclude that carrying capacity of each species changes in time and eventually converges along with the stability of the co-existence equilibrium.

INTRODUCTION

Rhinoceros sondaicus or better known as Javan rhino is one of the five rhinoceros that still exist in the world. According to the International Union for Conservation of Nature (IUCN) Red-List [3], the current status of Javan rhino is critically endangered. In addition, the Javan rhino is also listed in Appendix I of CITES (Convention on International Trade in Endangered Species of Wild Fauna and Flora) because of their numbers in the wild have been very few and feared to be extinct [8]. Nowadays, the Javan rhinos only exist in Indonesia at Ujung Kulon National Park. Ujung Kulon National Park as the last habitat for Javan rhino is a conservation area of 120,551 ha consisting of land covering area of 76,214 ha and water covering area of 44,337 ha [1].

Javan rhino population experienced the dynamics from year to year. Since 1980s the population is likely to remain at around 50. The results of monitoring in 2011 are identified there are 35 Javan rhinos consisting 22 males and 13 females. In 2012, 52 Javan rhinos are found with 29 males and 23 females. While the results of monitoring in 2013 reported that there are eight offsprings that bringing to the total 60 rhinos. Not very long from that time, still in 2013, the number of Javan rhinos remaining 58 because two rhinos reported was dead. The latest data show that the Javan rhino population consist of 35 males and 23 females, or 50 adult rhinos and eight children [6]. Javan rhino population is relatively small with high risk of extinction. Because of that fact, the efforts to ensure the preservation of Javan rhino population for a long time period is one of the priorities of conservation program in Indonesia.

Various source of information stated that many factors, both biotic factors and abiotic factors may cause the Javan rhino population declined. One of the factors is the threat of wild bull (Bos Javanicus) that also live in Ujung Kulon Natinal Park. Although these bull are not consuming the same food as rhino, but they number is approximately to 800 [6]. Therefore, in this article we concern of dynamical analysis of Javan rhino and bull population in Ujung Kulon National Park. It is assumed that both of rhino and bull grow logistically where the carrying capacity of both species is a function of rhino and bull.

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MATHEMATICAL MODEL

The logistic model in this paper modifies carrying capacity as the function of Javan rhino and bull. Hence, the model that we concern is

$$\begin{aligned}\frac{dR}{dt} &= \mu_r R\left(1-\frac{R}{C}\right)\\ \frac{dB}{dt} &= \mu_b B\left(1-\frac{B}{K}\right)\end{aligned} \tag{1}$$

where R represents rhino population, B is bull population, μ_r is intrinsic growth rate of rhino and μ_b is intrinsic growth rate of bull, C and K represent carrying capacity of rhino and bull respectively. The carrying capacity function is given as follow

$$\begin{aligned}C &= C_0\left(1-\frac{B}{\alpha+B}\right)\\ K &= K_0\left(1-\frac{R}{\beta+R}\right)\end{aligned} \tag{2}$$

where C_0 and K_0 are initial carrying capacity of rhino and bull respectively, α is the influence of bull existence to the rhino carrying capacity, and β is the influence of rhino existence to the bull carrying capacity. Hence, the modified carrying capacity model is given below

$$\begin{aligned}\frac{dR}{dt} &= \frac{\mu_r R\left(C_0\alpha - R\alpha - RB\right)}{C_0\alpha}\\ \frac{dB}{dt} &= \frac{\mu_b B\left(K_0\beta - B\beta - RB\right)}{K_0\beta}\end{aligned} \tag{3}$$

The system (3) is the model that we concern for finding the equilibrium points and doing some numerical simulation in the next chapter.

EQUILIBRIA AND LOCAL STABILITY ANALYSIS

The possible equilibrium points of system (3) are $E_1(0,0)$, $E_2(C_0,B)$ and $E_3(0,K_0)$. By doing elimination between $\frac{dR}{dt}$ and $\frac{dB}{dt}$ obtained

$$B = \frac{K_0\beta}{\beta+R}\text{, and} \tag{4}$$

$$\alpha R^2 + \left(\alpha\beta - \alpha C_0 + \beta K_0\right)R - \alpha\beta C_0 \tag{5}$$

The general Jacobian matric for the system (3) is

$$J = \begin{pmatrix} \dfrac{\mu_r\left(C_0\alpha - 2\alpha R - 2RB\right)}{C_0\alpha} & -\dfrac{R^2\mu_r}{C_0\alpha} \\ -\dfrac{B^2\mu_b}{K_0\beta} & \dfrac{\mu_b\left(K_0\beta - 2\beta B - 2RB\right)}{K_0\beta}\end{pmatrix} \tag{6}$$

Substituting the equation (4) to the Jacobian matric then we will be obtained a new Jacobian matric such that

$$J_B = \begin{pmatrix} \dfrac{\mu_r\left(C_0\alpha\beta + C_0\alpha R - 2\alpha\beta R - 2\alpha R^2 - 2\beta RK_0\right)}{C_0\alpha\left(\beta+R\right)} & -\dfrac{R^2\mu_r}{C_0\alpha} \\ -\dfrac{\beta K_0\mu_b}{\left(\beta+R\right)^2} & -\mu_b \end{pmatrix} \tag{7}$$

Furthermore, from (5)

$$K_0 = \frac{\alpha\left(C_0\beta + C_0 R - \beta R - R^2\right)}{\beta R} \tag{8}$$

Hence, the Jacobian matrix after substituting equation (4) and (8) is given by

$$J^* = \begin{pmatrix} -\mu_r & -\dfrac{R^2\mu_r}{C_0\alpha} \\ -\dfrac{\mu_b\left(C_0 - R\right)}{\left(\beta+R\right)R} & -\mu_b \end{pmatrix} \tag{9}$$

The characteristic equation from Jacobian matrix (9) is

$$C_0\left(\beta+R\right)\lambda^2 + C_0\left(\mu_b+\mu_r\right)\left(\beta+R\right)\lambda + \mu_r\mu_b\left(C_0\beta + R^2\right) \tag{10}$$

This equation has two eigenvalues which is λ_1 and λ_2. It shows that $\lambda_1 < 0$ and $\lambda_2 < 0$ so the system (3) is stable with no condition where $\lambda_1 + \lambda_2 < 0$ and $\lambda_1\lambda_2 > 0$.

NUMERICAL SIMULATION

Numerical simulations are provided to illustrate the stability of the system and the dynamics of coexistence from the two populations. It is performed by taking $C_0 = K_0 = 1000, \alpha = \beta = 200, \mu_b = 0.05$ and $\mu_r = 0.02$.

In figure 1 we choose some different initial conditions for rhino and bull population. The simulation shows that every initial condition will be stable to one point, which is stable equilibrium point namely $E^* = (377.72, 343.17)$. The number in E^* in such an unfixed number, but the equilibrium point will be around that. It means, when rhino population reach that number ($R = 377$), the population of bull will be around 343. Here, the carrying capacity of both population made to the same number as well as the influenced of both species.

Figure 2 shows the phase portrait for dynamical simulation that occurred between two populations. With the same data as the recent figure, it has given the explanation that both rhino and bull will be coexisted for such a long period of time. The second simulation is conducted by taking the data with respect to α and β because those parameters have an effect to each population.

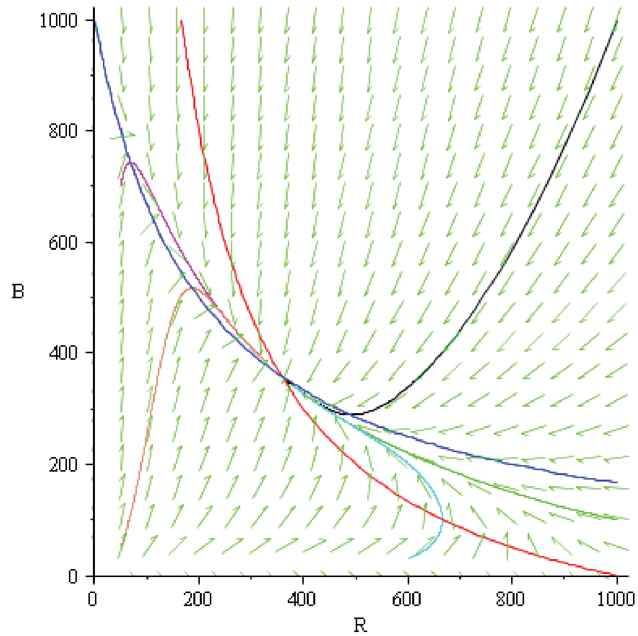

FIGURE 1. Phase portrait of system (3) with $C_0 = K_0 = 1000, \alpha = \beta = 200, \mu_b = 0.05$ and $\mu_r = 0.02$ by taking different initial conditions.

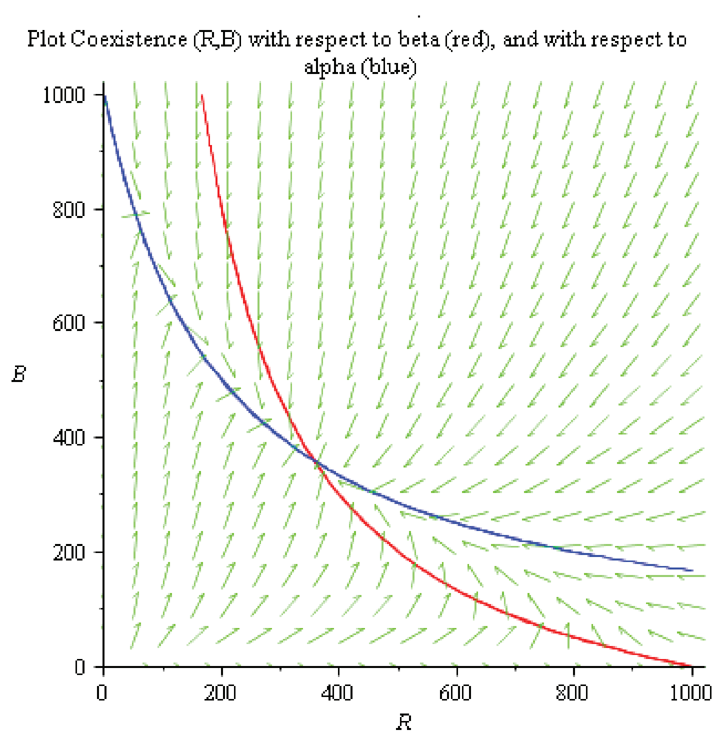

FIGURE 2. Phase portrait of coexistence between rhino and bull population

CONCLUSION

The logistic model for Javan rhino and bull population in Ujung Kulon National Park is a system of two-dimensional nonlinear ordinary differential equation [4]. The system has three equilibrium points namely E_1, E_2, and E_3, including the trivial one. Based on the analysis, the system is stable with no condition. Moreover, numerical simulations showed the stability of the system and the coexistence diagram between two populations.

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