

# The Economics of Synthetic Rhino Horns\*

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## Abstract

To examine the potential impact of synthetic horns to reduce rhino poaching, a formal model of the rhino horn market in which there exist firms with the capability to produce high quality synthetic horns is presented and studied. The analysis shows that whether the availability of synthetic horns would decrease the equilibrium supply of wild horns—and how much the reduction would be—depends on market structure—i.e., how competitive the synthetic horn production sector is—and on how substitutable the synthetic horns are for wild horns. The implications of these results for conservation policies are derived and discussed. Synthetic horn producers would benefit more by promoting their products as being superior to wild horns, but this could increase horn prices and lead to more rhino poaching. For conservation purposes, it may be beneficial to incentivize firms to produce inferior fakes—synthetic horns that are engineered to be undesirable in some respect but difficult for buyers to distinguish from wild horns. The analysis also shows that promoting competition in the production of synthetic horns in general is desirable from a conservation standpoint as synthetic horn producers may prefer to keep prices at a high enough level that could still encourage significant amount of poaching.

**Key words:** conservation; rhinos; rhino horns; poaching; synthetic horns; fake horns; adverse selection

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# 1 Introduction

Rhino horns are highly valued in Asian countries such as Vietnam, and the rhino populations in Africa and Asia continue to be threatened by poaching [Biggs et al, 2013; UNEP, 2016]. Poachers have become more sophisticated in recent years in terms of skill and equipment [UNEP, 2016]—no doubt a reflection of how lucrative trading in rhino horns is. In terms of price per unit weight, rhino horns are worth more than gold or diamond [Biggs et al, 2013].

To help deal with the poaching crisis, a few biotech companies have set their sights on developing synthetic versions of rhino horns using the latest science and technology [Corbyn, 2015]. The premise behind such a strategy is straightforward: if synthetic horns that are biologically identical (bio-identical) to the real thing can be produced at a lower cost compared to the cost of supplying wild horns, the demand for wild horns would decrease as buyers shift consumption towards the synthetic products. This, of course, would reduce people's incentive to poach rhinos.

The idea of using synthetic horns as a form of anti-poaching measure is a controversial one in the conservation community. Major rhino conservation groups are strongly opposed to it for they fear that the availability of synthetic horns would actually increase the demand for wild horns—by, e.g., lending legitimacy to the rhino horn trade—and exacerbate the poaching problem [Save the Rhino International & International Rhino Foundation, 2015].

One reason why those who are concerned about the fate of the rhinos are locked in the debate over how beneficial synthetic horns would be as an anti-poaching solution is that there appears to have been little, if any, rigorous analysis of this issue based on formal economic theory. Further, beyond the question of whether synthetic horns would benefit or hurt the rhino population, there has been scant discussion about what would be the most effective way—in terms of reducing the supply of wild horns—to utilize the technological capability to produce high quality synthetic horns, and whether there are policies that can be implemented by governments or conservation groups that can enhance the potential of this technology to curtail rhino poaching. While biotech firms seem keen on producing bio-identical synthetic horns [Corbyn, 2015], what would happen to wild horn supply if these companies were to sell synthetic horns that are designed to be undesirable in some respect yet difficult for buyers to distinguish from the real horns? How

does asymmetric information about horn quality affect the impact that synthetic horns can have on wild horn trading?

To examine these issues rigorously, a model of the horn market in which there are profit maximizing firms with the capability to produce high quality synthetic horns is presented and analyzed. Due to the possible existence of asymmetric information in the market, the model here builds on Akerlof's famous model of the used car market [Akerlof, 1970]. While the market in Akerlof's model is perfectly competitive, the model here allows for imperfect competition since biotech companies with the capability to produce synthetic horns may possess some market power over horn prices.

The analysis shows that whether the availability of synthetic horns would decrease the equilibrium supply of wild horns—and how much the reduction would be—depends on market structure—i.e., how competitive the synthetic horn production sector is—and on how substitutable the synthetic horns are for wild horns. The implications of these results for conservation policies are derived and discussed. Synthetic horn producers would benefit more by promoting their products as being superior to wild horns, but this could increase horn prices and lead to more rhino poaching. For conservation purposes, it may be beneficial to incentivize firms to produce inferior fakes—synthetic horns that are engineered to be undesirable in some respect but difficult for buyers to distinguish from wild horns. The analysis also shows that promoting competition in the production of synthetic horns in general is desirable from a conservation standpoint as synthetic horn producers may prefer to keep prices at a high enough level that could still encourage significant amount of poaching.

The exposition is organized as follows. In Section 2, a model of the horn market is presented. In Section 3, the concept of an equilibrium of the market is defined formally and its existence is established. The main results of this paper are given in Section 4, which looks at how the presence of synthetic horn producers impacts the supply of wild horns in the model. The policy implications of the analysis for rhino conservation are discussed in Section 5. Section 6 concludes.

## 2 The Model

Suppose there are two types of goods in the market for rhino horns: wild horns, and synthetic horns. The market consists of: a continuum of buyers; a continuum of sellers of wild horns; and a monopoly producer of synthetic horns. (A discussion of alternative market structure is deferred to Section 5, which examines how the level of competition in the production of synthetic horns affects the wild horn supply.)

### 2.1 Sellers of Wild Horns

Each seller of wild horns can procure and sell one unit at cost  $c > 0$ . This cost can differ across sellers: the distribution of  $c$  among the sellers is given by the cumulative distribution function  $F(\cdot)$ . The measure of these sellers is  $N_S > 0$ . For simplicity, assume that  $F(\cdot)$  is continuous. Further, assume that there is some minimum cost  $\underline{c} > 0$  of supplying wild horns so that  $F(c) = 0$  for all  $c \leq \underline{c}$  and  $F(c) > 0$  for all  $c > \underline{c}$ .

All wild horn sellers are price-takers: given price  $p$  for horns, a wild horn seller chooses whether to supply one unit of the good or not. Therefore, for a seller with cost  $c$ , it is profit maximizing to supply one unit of wild horn if  $p \geq c$ , and not to sell any otherwise. With the given distribution function for  $c$ , the total supply of wild horns at price  $p$  is thus  $N_S F(p)$ .

### 2.2 Monopoly Producer of Synthetic Horns

The monopoly can produce any non-negative amount  $Q$  of synthetic horns at cost  $C(Q)$ , where  $C(\cdot)$  is an increasing, continuous, and differentiable function. The monopoly's objective is to maximize its profit by choosing  $Q$ . Assume that the monopoly has a cost advantage over the wild horn sellers in the sense that the monopoly's marginal cost of producing the  $q$ -th unit of synthetic horn is lower than the marginal cost of supplying the  $q$ -th unit of wild horn for all  $q$ .

### 2.3 Buyers

The measure of buyers is  $N_B > 0$ . The synthetic horns are of high quality in the sense that buyers cannot distinguish between the two types of horns (equivalently, the cost of testing or differentiating between the two is prohibitive). The value of the wild horn to a buyer is  $V_W \geq 0$ , while the value of the synthetic horn

is  $V_S \geq 0$ . The distribution of  $V_W$  among the buyers is given by the continuous cumulative distribution function  $G(\cdot)$ . To avoid trivialities, assume that  $G(\underline{c}) < 1$  so that wild horns would be traded in the market if no synthetic horns are available. For convenience, let us refer to a buyer whose valuation of the wild horn is  $V_W$  as a type- $V_W$  buyer. Assume that the value of the synthetic horn to a type- $V_W$  buyer is given by  $h(V_W)$ , i.e.,  $V_S = h(V_W)$ , where  $h$  is some non-negative, strictly increasing, and continuous function.

**Remark 1** *The model can be generalized to accommodate a wider range of relationship between a buyer's valuation of the wild horn  $V_W$  and the buyer's valuation of the synthetic horn  $V_S$ . This can be done by specifying the distribution of  $V_S$  for each value of  $V_W$ , i.e., how the buyers' preference for the synthetic horn is distributed in the population as a function of their valuation of the wild horn. However, such a generalization is not necessary for the main points of this paper, and the assumption that  $V_S$  is monotonically and continuously related to  $V_W$  is adopted here to simplify the proof of the existence of a market equilibrium (Theorem 4).*

Given that buyers cannot distinguish between wild horns and the synthetic substitutes, there is only one price in the market for horns. Buyers are price-takers: given price  $p$  for horns, a buyer chooses whether to buy one unit or not to buy any. Because buyers cannot observe the type of horns sold in the market, they need to consider the expected value of the horns for sale when they make their purchase decision.

Letting  $\pi$  denote the probability that a horn for sale is wild, the expected value of a horn available for purchase to a type- $V_W$  buyer is  $\pi V_W + (1 - \pi) V_S = \pi V_W + (1 - \pi) h(V_W)$ . Hence, given price  $p$ , it is optimal for the buyer to purchase a unit if  $\pi V_W + (1 - \pi) h(V_W) \geq p$ , and not buy otherwise. Therefore, the total demand for horns at price  $p$  is given by  $N_B (1 - G(\underline{V}(p)))$ , where  $\underline{V}(p) \equiv \min \{V \in \mathbb{R}_+ \mid \pi V + (1 - \pi) h(V) \geq p\}$ . Given price  $p$ ,  $\underline{V}(p)$  is the threshold valuation of wild horn that determines whether a buyer would purchase a horn or not: a buyer whose valuation of the wild horn exceeds  $\underline{V}(p)$  would choose to buy, while a buyer whose valuation of the wild horn is below this threshold would not make a purchase. Note that, because  $h(\cdot)$  is strictly increasing by assumption, the set  $\{V \in \mathbb{R}_+ \mid \pi V + (1 - \pi) h(V) \geq p\}$  is non-empty for any  $\pi \in [0, 1]$  and  $p \geq 0$ .

Assuming that the buyers know the distribution of cost among the wild horn suppliers, the belief  $\pi$  must be the proportion of wild horns among all the horns being sold in the market, i.e.,

$$\pi = \pi(p, Q) = \frac{N_S F(p)}{N_S F(p) + Q}. \quad (1)$$

## 2.4 Market Equilibrium

In an equilibrium of the market, the price of horns must clear the market—the number of units that buyers want to buy at the equilibrium price must equal the number of units supplied by the wild horn sellers and the monopoly synthetic horn producer.

To derive the monopoly's production of synthetic horns, we first need to determine the relationship between  $Q$ , the monopoly's quantity, and the market price of horns  $p$ . Recall that, given price  $p$ , the demand for horns is  $N_B (1 - G(\underline{V}(p)))$ , and the total supply of horns is  $N_S F(p) + Q$ . Fixing  $Q$ , let  $P(Q)$  denote a price  $p \geq 0$  that satisfies the market clearing condition

$$N_B (1 - G(\underline{V}(p))) = N_S F(p) + Q. \quad (2)$$

Equation (2) allows us to implicitly define a market clearing price function  $P(\cdot)$  that specifies how the market price of horns varies with the monopoly's production of synthetic horns.

Given the price function  $P(\cdot)$ , the monopoly's profit maximization problem is

$$\max_{Q \geq 0} \{P(Q) Q - C(Q)\}. \quad (3)$$

A market equilibrium of the model can then be defined by a quantity-price pair  $(Q^*, p^*)$  such that: given  $P(\cdot)$ ,  $Q^*$  is profit maximizing for the monopoly; and  $p^* = P(Q^*)$  clears the market.

**Remark 2** *Since the demand for horns is at most  $N_B$ , if the monopoly chooses  $Q > N_B$ , then the monopoly would be saddled with unsold units, which is clearly not profit maximizing. Therefore, it can be assumed henceforth that  $Q \in [0, N_B]$ .*

### 3 Existence of Market Equilibrium

In this section, the question of whether a market equilibrium always exists will be addressed.

Without the monopoly producer of synthetic horns in the market, the equilibrium price of horns solves (2) with  $Q$  set at 0. In this case,  $\underline{V}(p) = p$ , and equation (2) must have a solution by the Intermediate Value Theorem.

With the monopoly in the market, it can be shown that a solution to (2) must exist for any  $Q \in [0, N_B]$  (see Lemma 15 in the Appendix). If  $P(Q)$  is unique for all  $Q \geq 0$ , then it must be continuous in  $Q$ . This, along with the restriction  $Q \in [0, N_B]$ , imply that the monopoly's optimization problem (3) has a solution; hence, a market equilibrium must exist in this case.

Equation (2), however, can have multiple solutions, i.e., given some  $Q \in [0, N_B]$ , there can be more than one market clearing price (see the Appendix for an example). The intuition for this multiplicity is straightforward and is the same as that for the possible coexistence of multiple equilibria in the canonical Akerlof's lemons market (in which all sellers are price-takers). Suppose  $V > h(V)$  so that synthetic horns are viewed by buyers as being inferior to wild horns. Consider the effect on demand for horns when price increases. From the buyers' perspective, a high price, *ceteris paribus*, lowers consumer surplus and thus reduces demand. However, there is a second, countervailing effect at work: given  $Q > 0$ , a high horn price leads to a large supply of wild horns, which—as can be seen from (1)—increases the fraction of wild horns in the market; and when horn prices are high, the greater probability of obtaining a horn that is wild increases buyers' willingness-to-pay, which gives buyers more incentive to buy horns. If the latter effect (which tends to increase demand) more than offsets the first effect (which tends to decrease demand), then an increase in horn price, fixing  $Q$ , would raise both the supply of horns and the demand for horns, making it possible for the market to clear at a high price.

Using an analogous argument, a decrease in horn price, by reducing the supply of wild horns and thus the fraction of horns for sale that are wild, can result in both lower supply of horns and lower demand for horns. This gives rise to the possibility of the market also being able to clear at a relatively low price.

The fact that there can be multiple market clearing prices fixing  $Q$  raises the question of how  $P(Q)$  is to be defined when equation (2) has more than one solution given  $Q$ . Since all sellers in the model are

profit maximizers who would like to receive the highest price they can get for their product, in the following analysis  $P(Q)$  is defined to be the highest price that clears the market if the monopoly produces  $Q$  units of synthetic horns. This allows a market equilibrium of the model to be defined precisely as follows.

**Definition 3** *Let*

$$P(Q) \equiv \sup \{p \in \mathbb{R}_+ \mid N_B(1 - G(\underline{V}(p))) = N_S F(p) + Q\}. \quad (4)$$

*A **market equilibrium** is a quantity-price pair  $(Q^*, p^*)$  satisfying:*

1.  $Q^* \in \arg \max_{Q \geq 0} \{P(Q)Q - C(Q)\};$
2.  $p^* = P(Q^*).$

If there are values of  $Q$  for which multiple market clearing prices exist, then  $P(Q)$  as defined in (4) need not be continuous in  $Q$ . Despite the possibility of discontinuity, defining  $P(Q)$  to be the highest market clearing price given  $Q$  ensures that a solution to the monopoly's profit maximization problem—and, hence, a market equilibrium—exists.

**Theorem 4** *A market equilibrium exists.*

**Proof.** See the Appendix. ■

**Remark 5** *Lemma 17 in the Appendix shows that the sup in (4) can be replaced with max.*

## 4 Effect of Synthetic Horns on the Supply of Wild Horns

In this section, the impact on the supply of wild horns if the monopoly producer of synthetic horns is able to enter the horn market will be considered.

### 4.1 Perfect Substitutes ( $h(V) = V$ )

Let us first consider the effect of the monopoly's production of synthetic horns on the supply of wild horns if the synthetic products are considered by buyers to be equivalent to the wild ones, i.e., the synthetic horns are bio-identical to and perfectly substitutable for wild horns. In this case,  $h(V) = V$ , which yields  $\underline{V}(p) = p$ .



Equation (2) then implies that  $P(Q)$  is strictly decreasing (see Lemma 19 in the Appendix). This yields the following result.

**Proposition 6** *If  $h(V) = V$ , the availability of the synthetic horns would reduce the equilibrium supply of wild horns. However, the suppliers of wild horns would not necessarily be driven out of the market.*

To see the first part of Proposition 6, note that the equilibrium price of horns in the absence of the monopoly is  $P(0)$ , which, given the model assumptions, is strictly greater than  $\underline{c}$ . When the monopoly is in the market, its marginal revenue at  $Q = 0$  is  $P(0)$ , and its marginal cost at  $Q = 0$  is  $C'(0)$ . Since  $P(0) > \underline{c} > C'(0)$ , where the second inequality follows from the assumed cost advantage that the monopoly has over the wild horn sellers, the monopoly would produce a strictly positive amount of synthetic horns, i.e.,  $Q^* > 0$ . Since  $P(\cdot)$  is strictly decreasing in this case,  $P(Q^*) < P(0)$ , which means the equilibrium supply of wild horns must be lower when synthetic horns are being sold in the market.

To prove the second part of Proposition 6, consider a specification of the model in which: (i)  $c$  is distributed uniformly over the interval  $[\underline{c}, \bar{c}]$  for some  $\bar{c} > \underline{c}$ ; (ii)  $V_W$  is distributed uniformly over the interval  $[0, \bar{V}]$ , where  $\bar{V} > \underline{c}$ ; and (iii)  $C'(Q)$ , the monopoly's marginal cost of producing synthetic horns, is constant at some  $\gamma > 0$ . To capture the fact that the synthetic horns are cheaper to supply than the wild horns, assume that  $\gamma < \underline{c}$ , i.e., the monopoly's marginal cost is lower than the minimum marginal cost of supplying wild horns.

With this specification,  $P(\cdot)$  is a piecewise linear function. Assuming that  $N_B(1 - (\bar{c}/\bar{V})) \leq N_S$ ,  $P(\cdot)$  is given by

$$P(Q) = \begin{cases} \frac{\bar{V}}{N_B(\bar{c}-\underline{c})+N_S\bar{V}} [(N_B - Q)(\bar{c} - \underline{c}) + N_S\underline{c}] & \text{if } Q \leq N_B(1 - (\underline{c}/\bar{V})) \\ \frac{\bar{V}}{N_B}(N_B - Q) & \text{if } Q > N_B(1 - (\underline{c}/\bar{V})) \end{cases}.$$

Now, find the value of  $Q$ —call it  $\bar{Q}$ —such that the price that clears the market is  $\underline{c}$ , i.e.,  $\bar{Q}$  solves  $P(\bar{Q}) = \underline{c}$ . This yields  $\bar{Q} = N_B(1 - (\underline{c}/\bar{V}))$ . Note that when  $Q < \bar{Q}$ ,  $P(Q) > \underline{c}$ , which means the supply of wild horns is non-zero; if  $Q \geq \bar{Q}$ , then  $P(Q) \leq \underline{c}$ , which would lead to no supply of wild horns.

The monopoly's revenue function is  $P(Q)Q$ , and its marginal revenue function is  $MR(Q) \equiv \frac{d(P(Q)Q)}{dQ}$ . It is easy to check that  $MR(\cdot)$  is strictly decreasing. At  $Q = \bar{Q}$ , there is a discontinuity in the monopoly's

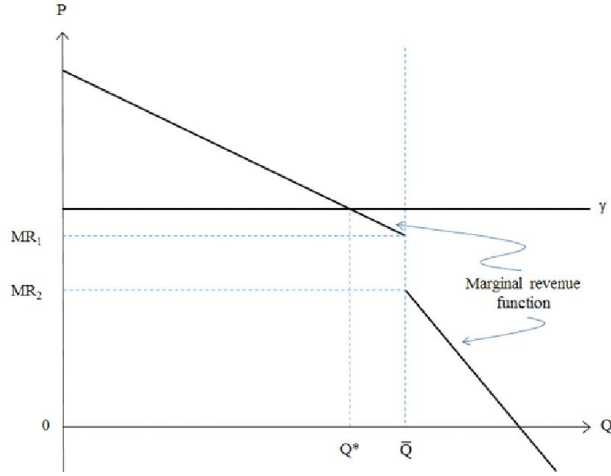


Figure 1: Graphical illustration of the model specification given in Section 4.1. If the monopoly's marginal cost  $\gamma$  is not sufficiently low, then there is a positive supply of wild horns in equilibrium.

marginal revenue function (since there is a kink in  $P(\cdot)$  at  $\bar{Q}$ ). The marginal revenue as  $Q$  approaches  $\bar{Q}$  from below is

$$MR_1 \equiv \frac{N_B (\bar{c} - \underline{c}) (2\underline{c} - \bar{V}) + N_S \underline{c} \bar{V}}{N_B (\bar{c} - \underline{c}) + N_S \bar{V}};$$

the marginal revenue as  $Q$  approaches  $\bar{Q}$  from above is

$$MR_2 \equiv 2\underline{c} - \bar{V};$$

and

$$MR_1 - MR_2 = \frac{N_S \bar{V} (\bar{V} - \underline{c})}{N_B (\bar{c} - \underline{c}) + N_S \bar{V}} > 0.$$

If  $\gamma > MR_1$ , then the monopoly's profit maximizing quantity is below  $\bar{Q}$ , and there is a strictly positive supply of wild horns in equilibrium; if  $\gamma \leq MR_1$ , then the monopoly's profit maximizing quantity is equal to or above  $\bar{Q}$ , and the equilibrium supply of wild horns is zero. See Figure 1 for a graphical illustration of this model specification.

The implications of Proposition 6 are clear. Compared to the case in which only wild horns are sold in the market, the availability of the synthetic horns increases the market supply of horns, which reduces their price, and this decreases the amount of wild horns in the market. However, even if the synthetic horns can

be supplied at a lower cost than wild horns, there is no guarantee that wild horn sellers would be driven out of business since profit maximization for the monopoly in this case does not necessarily mean that it should drop the price low enough to get rid of all competition coming from wild horn sellers. Depending on parameter values, the monopoly producer of synthetic horns may find it more profitable to keep the market price of horns not too low—i.e., above  $\underline{c}$ —by producing a low amount of synthetic horns than to flood the market with its products and drive down the price enough to force out all wild horn sellers.

## 4.2 Superior Substitutes ( $h(V) > V$ )

It appears that some of the companies interested in the development and production of synthetic horns would opt for an ‘overt’ marketing strategy that touts their synthetic products as being just as good if not better than wild horns (due to, for example, lack of contaminants in the synthetic products) [Broad & Burgess, 2016]. As Proposition 7 below states, even if synthetic horns can be supplied at a lower cost than wild horns, this type of product promotion and marketing can backfire from a conservation viewpoint: if buyers find synthetic horns to be more desirable than wild horns, then the presence of the monopoly in the market can actually increase the equilibrium price of horns and lead to a greater supply of wild horns.

**Proposition 7** *If buyers find synthetic horns to be better than wild horns ( $h(V) > V$ ), then having the monopoly producer of synthetic horns in the market can increase the equilibrium supply of wild horns.*

This result is shown using the following example.

**Example 8** *Suppose  $V_W$  is distributed uniformly over  $[0, 100]$ ,  $c$  is distributed uniformly over  $[\underline{c}, \underline{c} + 100]$  for some  $\underline{c} \in (0, 100)$ , and the monopoly’s marginal cost is constant at  $\gamma < \underline{c}$ . Let  $N_B = N_S = 100$ , and assume that  $h(V) = \theta V$  for some constant  $\theta > 1$ . Given  $\underline{c} = 50$  and  $\theta = 5$ , it is easy to show that the market clearing condition (2) has a unique solution for each  $Q$ : the price function  $P(\cdot)$  is given by*

$$P(Q) = \begin{cases} \frac{125}{2} - \frac{7}{4}Q + \frac{1}{4}\sqrt{9Q^2 + 2900Q + 2500} & \text{if } Q \leq 90 \\ 500 - 5Q & \text{if } Q > 90 \end{cases}.$$

*A plot of  $P(\cdot)$  is given in Figure 2. Note that the upward-sloping portion of  $P(\cdot)$  implies that it is possible to*

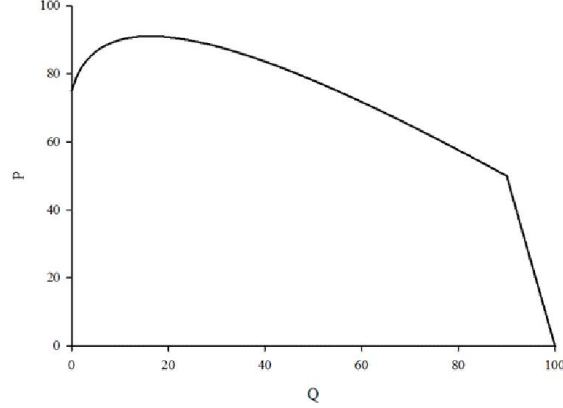


Figure 2: A plot of the price function  $P(\cdot)$  in Example 8.

specify a cost function for the monopoly such that the market equilibrium with the monopoly yields more wild horns than the equilibrium without the monopoly in the market. The monopoly's marginal revenue function given  $P(\cdot)$  is shown in Figure 3. If the monopoly's marginal cost is constant at  $\gamma = 48$ , then the monopoly's profit maximizing quantity  $Q^*$  is 50.1, and the market equilibrium price is  $P(Q^*) = 78$ . At this price, the supply of wild horns is 28. In contrast, if the monopoly is not in the horn market, then the equilibrium market price of horns is  $P(0) = 75$ , and the supply of wild horns is 25, i.e., the supply of wild horns is lower without the monopoly producer of synthetic horns.

Intuitively, when synthetic horns are considered by buyers to be better than the real thing, then having a synthetic horn producer in the market raises the average quality of goods, which—all else being equal—increases buyers' willingness-to-pay for horns. This puts upward pressure on the price of horns, which can result in a greater supply of wild horns.

### 4.3 Inferior Substitutes ( $h(V) < V$ )

What would happen to the equilibrium supply of wild horns if buyers do not consider bio-identical synthetic horns to be perfect substitutes for wild horns and thus would not be willing to pay as much for them as they would for the real thing? Similarly, what if the monopoly deliberately chooses to produce synthetic horns that are known to be undesirable in some respect (e.g., engineered to contain poison or toxic substances) yet

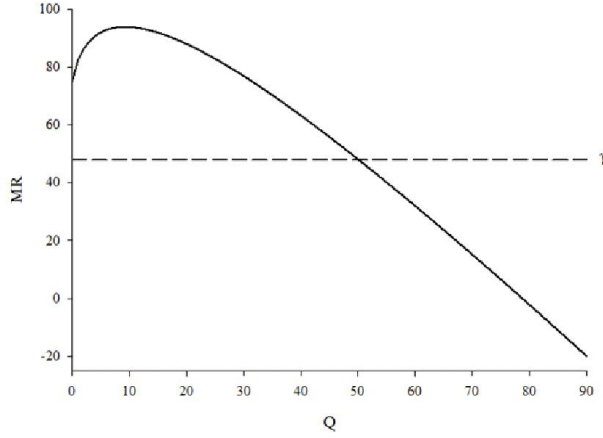


Figure 3: A plot of the monopoly's marginal revenue function in Example 8 for  $Q \leq 90$ . If the monopoly's marginal cost is constant at 48, then the monopoly's profit maximizing quantity is 50.1.

bio-similar to wild horns so that they are not easily differentiated from them?

To answer these questions, let us now compare the impact of synthetic horns on the horn market when the monopoly's product is viewed as being perfectly substitutable for wild horns ( $h(V) = V$ ) to the effect of synthetic horns when they are valued less than wild horns by buyers ( $h(V) < V$ ). For this purpose, let  $h_1(\cdot)$  denote the identity function (i.e.,  $h_1(V) = V$  for all  $V$ ) and pick a function  $h_2(\cdot)$  such that  $h_2(V) < V$  for all  $V$ . Now, define the following: for  $i = 1, 2$ ,

$$\underline{V}_i(p) \equiv \min \{V \in \mathbb{R}_+ \mid \pi V + (1 - \pi) h_i(V) \geq p\}$$

and

$$P_i(Q) \equiv \max \{p \in \mathbb{R}_+ \mid N_B(1 - G(\underline{V}_i(p))) = N_S F(p) + Q\}.$$

In words,  $P_1(Q)$  is the (highest) market clearing price of horns when the monopoly produces  $Q$  units of perfect substitutes, and  $P_2(Q)$  is the (highest) market clearing price of horns when the monopoly produces  $Q$  units of inferior substitutes.

**Proposition 9**  $P_2(Q) \leq P_1(Q)$  for all  $Q \in [0, N_B]$ .

**Proof.** First, it is clear that, given  $p$ ,

$$\{V \in \mathbb{R}_+ \mid \pi V + (1 - \pi) h_2(V) \geq p\} \subset \{V \in \mathbb{R}_+ \mid \pi V + (1 - \pi) h_1(V) \geq p\}.$$

Hence, it must be the case that

$$\min \{V \in \mathbb{R}_+ \mid \pi V + (1 - \pi) h_2(V) \geq p\} \geq \min \{V \in \mathbb{R}_+ \mid \pi V + (1 - \pi) h_1(V) \geq p\},$$

i.e.,  $\underline{V}_2(p) \geq \underline{V}_1(p)$ .

Now, we know that  $N_B(1 - G(\underline{V}_1(P_1(Q)))) - (N_SF(P_1(Q)) + Q) = 0$  because  $P_1(Q)$  is a market clearing price given  $Q$ . Moreover, because  $P_1(Q)$  is the largest market clearing price, Lemma 14 in the Appendix implies that

$$N_B(1 - G(\underline{V}_1(p))) - (N_SF(p) + Q) < 0 \text{ for all } p > P_1(Q). \quad (5)$$

Since  $\underline{V}_2(p) \geq \underline{V}_1(p)$  for all  $p$  and  $G(\cdot)$  is an increasing function,  $N_B(1 - G(\underline{V}_2(p))) \leq N_B(1 - G(\underline{V}_1(p)))$  for all  $p$ , which together with (5) imply that  $N_B(1 - G(\underline{V}_2(p))) - (N_SF(p) + Q) < 0$  for all  $p > P_1(Q)$ . Therefore,  $P_2(Q) \not> P_1(Q)$ , i.e.,  $P_2(Q) \leq P_1(Q)$ . ■

A straightforward but significant implication of Proposition 9 is that the monopoly can generate more revenue by producing perfect substitutes rather than inferior substitutes. Therefore, given a choice between making synthetic horns that are viewed as being perfectly substitutable for wild horns and making synthetic horns that are inferior to wild horns in some respect but hard to distinguish from the real ones, a profit maximizing monopoly—all else equal—would choose to make perfect substitutes.

**Remark 10** *Using an analogous argument, the (highest) market clearing price of horns when the monopoly produces  $Q$  units of superior substitutes ( $h(V) > V$ ) must be at least as high as the (highest) market clearing price of horns when the monopoly produces  $Q$  units of perfect substitutes ( $h(V) = V$ ). Therefore, the monopoly can generate more revenue by promoting its product as being better than wild horns rather than being perfectly substitutable.*

It should be noted that even though the market price of horns is lower given any quantity of synthetic horns produced by the monopoly when the synthetic horns are inferior to wild horns rather than perfect substitutes, this does not necessarily imply that the equilibrium supply of wild horns is lower when the monopoly produces inferior substitutes. Relative to the case in which the monopoly produces perfect substitutes, the supply of wild horns may be higher in equilibrium when the monopoly produces inferior substitutes. This is shown by Example 11.

Intuitively, because the production of inferior substitutes tends to drive down horn price by reducing buyers' willingness-to-pay for horns, the monopoly producer of synthetic horns may compensate for this negative effect on revenue by keeping production low so as to prop up price. This effect can yield an outcome in which the equilibrium price is actually higher when the synthetic horns are inferior substitutes for wild horns than when the synthetic horns are perfect substitutes.

**Example 11** Suppose  $N_B = N_S = 100$ , buyer valuation of wild horns  $V_W$  is uniform over  $[0, 100]$ , and the cost of supplying wild horns  $c$  is uniform over  $[40, 140]$ . Assume the monopoly's marginal cost of producing synthetic horns is constant at 30 whether the monopoly produces synthetic horns that are perfect substitutes for wild horns or inferior substitutes.

With perfect substitutes, the price function  $P(\cdot)$  is

$$P(Q) = \begin{cases} 70 - \frac{1}{2}Q & \text{if } Q \leq 60 \\ 100 - Q & \text{if } Q > 60 \end{cases}. \quad (6)$$

The monopoly's marginal revenue function is  $70 - Q$  for  $Q \leq 60$ , and it is  $100 - 2Q$  for  $Q > 60$ . With the given marginal cost function, the monopoly's profit maximizing quantity is  $Q^* = 40$ , and the equilibrium market price is 50.

For the case with inferior substitutes, suppose  $h(V) = \frac{1}{10}V$ . The price function  $P(\cdot)$  in this case is

$$P(Q) = \begin{cases} 55 - \frac{21}{40}Q + \frac{1}{40}\sqrt{361Q^2 - 49200Q + 360000} & Q \leq 7.8 \\ 10 - \frac{1}{10}Q & Q > 7.8 \end{cases}.$$

Given a constant marginal cost of 30, the monopoly's profit maximizing quantity is  $Q^* = 7.05$ , and the

*equilibrium market price is 55.7.*

## 5 Implications for Conservation Policy

As mentioned in the Introduction, a handful of biotech companies seek to develop synthetic horns that are bio-identical to wild horns. The analysis here shows that the availability of synthetic horns that are perfectly substitutable for wild horns and can be supplied at a lower cost will drive down horn prices and decrease the amount of wild horns in the market (Proposition 6). This, of course, should benefit the rhino population by reducing hunters' incentive to poach rhinos for their horns.

There is, however, a key issue that has received little attention in the conservation community concerning the use of synthetic horns as a rhino conservation tool: if we have the technological capability to produce synthetic horns that are extremely difficult for buyers to distinguish from wild horns, what would be the most effective way to utilize this technology to reduce rhino poaching? Is it to produce bio-identical synthetic horns? Or should synthetic horn producers, taking advantage of information asymmetry in the market, make synthetic horns that buyers would consider vastly inferior to wild horns?

One disadvantage of producing bio-identical synthetic horns—as shown by Proposition 7—is that, if buyers view them to be more desirable than wild horns due to their lack of contaminants or because presumably no animals are harmed in producing them, their availability in the market could actually raise the price of horns and result in a greater supply of wild horns (and, hence, more poaching). Even if buyers consider bio-identical synthetic horns to be perfect substitutes for wild horns—i.e., no better and no worse—how effective synthetic horns are in reducing the supply of wild horns depends critically on market structure, specifically, how much competition there is on the production side of synthetic horns.

To illustrate this point, assume henceforth that bio-identical synthetic horns are perfect substitutes for wild horns. Recall from Proposition 6 that, because a monopoly has an incentive to keep prices high in order to boost profit, the availability of perfectly substitutable synthetic horns may not drive down prices enough to drive out wild horn suppliers when synthetic horns are produced by a single firm. Now, consider what would happen if there is competition in the synthetic horn sector and bio-identical synthetic horns are produced by multiple firms. Intuitively, the competition would put downward pressure on horn prices, leading to a lower



supply of wild horns compared to the market with a monopoly producer of synthetic horns. This implies that the following situation could occur: there is positive supply of wild horns in equilibrium when production of bio-identical synthetic horns is monopolized by a single firm; however, when there is perfect competition in the production of bio-identical synthetic horns, the equilibrium price is lowered sufficiently to drive out all wild horn producers. Example 12 shows this numerically using the model presented here.

**Example 12** *As in Example 11, suppose  $N_B = N_S = 100$ ,  $V_W$  is distributed uniformly over  $[0, 100]$ , and  $c$  is distributed uniformly over  $[40, 140]$ . Assume that synthetic horns are perfect substitutes for wild horns so that  $h(V) = V$ . If synthetic horns are produced by a monopoly, then the price function  $P(\cdot)$  is given by (6). If the monopoly's marginal cost function is  $10 + \frac{Q}{2}$ , then the monopoly's profit is maximized at  $Q^* = 40$ . In the market equilibrium, the horn price is 50 and the supply of wild horns is 10.*

*On the other hand, if the synthetic horn market is perfectly competitive, then the marginal cost function for producing synthetic horns given above is the supply curve for synthetic horns. The equilibrium price is one that equates the demand for horns to the total supply of horns (wild and synthetic). In this case, the equilibrium price of horns is 40, and all of the demand for horns is satisfied by synthetic horns, i.e., in equilibrium, there is no supply of wild horns.*

The effectiveness of using synthetic horns to reduce the supply of wild horns depends not only on market structure (as shown in Example 12), but also on what kind of synthetic horns are produced, specifically, whether the bio-fabricated horns are engineered to be bio-identical to wild horns or are made to be undesirable in some respect yet difficult to distinguish from wild horns. Why does the type of synthetic horns produced matter? When firms produce bio-identical synthetic horns that are viewed as perfect substitutes for wild horns, what buyers are willing to pay for horns is unaffected even if they cannot tell which horns are wild and which ones are synthetic, i.e., the presence of synthetic horns would have no effect on the demand curve for horns. However, if buyers are aware that there are undesirable fakes—which they place a low valuation on—in the market that they cannot distinguish from wild horns, their uncertainty over the quality of horns for sale would cause them to lower what they are willing to pay to acquire horns. And, compared to the case in which firms produce bio-identical synthetic horns, this would lead to a lower demand for horns.

To explore the implications of this further, consider the following example.

**Example 13** Again, suppose  $N_B = N_S = 100$ ,  $V_W$  is distributed uniformly over  $[0, 100]$ , and  $c$  is distributed uniformly over  $[40, 140]$ . The marginal cost function of producing synthetic horns is  $20 + Q$ .

**Perfect substitutes:** Suppose synthetic horns are perfect substitutes for wild horns ( $h(V) = V$ ), and suppose there is a monopoly producer of synthetic horns. The price function  $P(\cdot)$  is given by (6). It is easy to check that the monopoly's profit maximizing quantity is 25. The equilibrium market price is 57.5, and there is positive supply of wild horns in equilibrium (17.5).

If the synthetic horn market is perfectly competitive, then the synthetic horn supply curve is given by the marginal cost function specified above. In this case, the equilibrium quantity of horns is 46.7. The equilibrium price is 53.3, and, again, there is positive supply of wild horns in equilibrium (13.3).

**Inferior substitutes:** Suppose now synthetic horn suppliers only produce inferior substitutes ( $h(V) < V$ ). All else remain the same in the model. To be specific, suppose  $h(V) = \frac{V}{2}$ . If synthetic horns are produced by a monopoly, then the price function  $P(\cdot)$  is given by

$$P(Q) = \begin{cases} 55 - \frac{5}{8}Q + \frac{1}{8}\sqrt{9Q^2 - 880Q + 14400} & \text{if } Q \leq 20.8 \\ 50 - \frac{1}{2}Q & \text{if } Q > 20.8 \end{cases}.$$

The monopoly's profit maximizing quantity is  $Q^* = 14.6$ , and the price of horns in equilibrium is 53.2.

With perfect competition in the production of synthetic horns, there is a unique market equilibrium in which the price is 40 and all horns for sale are synthetic (no supply of wild horns).

As Example 13 makes clear, both the market structure of the synthetic horn production sector and the type of synthetic horns that are produced affect how much impact the availability of synthetic horns has on the supply of wild horns. In the example, when synthetic horns are perfect substitutes for wild horns, wild horn suppliers cannot be driven out of the market even when there is perfect competition in the production of synthetic horns. The situation is different when synthetic horns are inferior substitutes, however. In this case, while some wild horn suppliers will remain in the market when there is a monopoly synthetic horn producer, there is no wild horn supply with perfect competition in the production of synthetic horns.

A few key policy implications for rhino conservation that can be drawn from the analysis here are the following. First, to fully take advantage of the power of synthetic biology to save the rhinos and to maximize

the impact of synthetic horns on reducing wild horn supply, policies to promote competition in the production and sale of synthetic horns should be implemented. Financial incentives or regulatory changes that make it easier or more lucrative for firms to enter the synthetic horn market would put downward pressure on horn prices that makes rhino poaching and the selling of wild horns less profitable.

Second, subsidies to the makers of synthetic horns that incentivize them to increase their output and lower prices would benefit the rhinos by reducing the supply of wild horns. As illustrated by the example given in Section 4.1, if the marginal cost of producing bio-identical synthetic horns is sufficiently low, the presence of a monopoly producer of synthetic horns in the market would drive out wild horn sellers by reducing price to a point where it is not profitable to supply wild horns. Since subsidies can be used to lower producers' marginal cost, a suitably chosen subsidy to manufacturers of synthetic horns would eliminate wild horn supply from the market.

Third, because synthetic horn producers may keep horn prices at a high enough level to allow wild horn sellers to stay in the market (Proposition 6), and because synthetic horn producers prefer to make perfect substitutes rather than inferior substitutes (a corollary of Proposition 9), another implication of the analysis here is that policies or actions that make it less costly for non-profit organizations to acquire the technological capability to produce synthetic horns should be supported. Organizations that are not intent on profiting from the production of synthetic horns could make a high enough quantity of bio-identical synthetic horns to drive down prices sufficiently to cause wild horn suppliers to exit the market. While the production of a large quantity of bio-identical synthetic horns may be extremely costly, another strategy that a non-profit organization could pursue is to produce inferior substitutes. Because the presence of undesirable synthetic horns in the market creates uncertainty for buyers and lowers demand for horns, a non-profit entity may not need to produce a high amount of inferior substitutes—and, hence, incur a large cost—to drive out wild horn suppliers through adverse selection.

## 6 Discussion and Conclusion

Rhino poaching numbers are high and rising. This observation coupled with the fact that the economies of China and Southeast Asia—where most of the demand for rhino horns comes from—are projected to continue

to grow at a healthy rate [OECD, 2016] portend a grim future for the rhinos if no significant changes occur.

Economic principles tell us that the availability of synthetic horns can reduce the supply of wild horns—and even drive out wild horn sellers completely from the horn market. However, previous discussions of this topic rarely touch on the question of what major factors can impact the effectiveness of synthetic horns to decrease wild horn supply and what can be done to best utilize synthetic horns as a conservation tool.

The main lesson provided by the results here is that the market structure of the synthetic horn sector and the type of synthetic horns that are produced matter greatly in determining how much—and what kind of—effect the availability of synthetic horns has on wild horn supply. For this reason, the type of policies that are implemented in the synthetic horn market would play a pivotal role in helping us chart the future fate of the rhinos. The results here show how important it is that we have a clear understanding of the economics of synthetic horns if we care about the most effective way to save the rhinos.

On a technical note, because multiple market clearing prices can coexist fixing the quantity of synthetic horns in the market, the analysis here assumes that the market price is the highest price that clears the market. This assumption, however, is not overly restrictive in that the main results of the paper—and their implications—do not depend on it. For instance, aside from the results dealing with the existence of a market equilibrium, none of the examples, propositions, or lemmas presented here would be affected substantively if the market price were to be defined instead as the lowest price that clears the market. While the existence of an equilibrium cannot be guaranteed with this alternative definition of the market price (because there may not be a solution to the monopoly’s maximization problem (3)), this non-existence is simply a consequence of the assumption that the monopoly’s quantity can be any non-negative real number, and not the result of some fundamental flaw of the model. More specifically, if the monopoly’s choices of quantity are restricted to be non-negative integers, then an equilibrium can always be found even if the market price is defined to be the lowest price that clears the market.

It is important to point out that the analysis here assumes that the availability of synthetic horns does not alter how buyers value wild horns, i.e., the distribution of  $V_W$  in the model does not change with the introduction of synthetic horns. If the presence of synthetic horns somehow reduces the value that people place on wild horns, then the development of synthetic horns would have a greater potential to decrease

rhino poaching than in the situation examined here by directly dampening the demand for wild horns. On the other hand, if the availability of synthetic horns in the market raises buyers' valuation of wild horns—by, for instance, changing how people perceive the legitimacy of horn consumption—then the production and sale of synthetic horns would be more likely to yield perverse effects on conservation efforts and increase wild horn supply. If and how the presence of bio-similar or bio-identical synthetic horns would affect people's valuation of wild horns is ultimately an empirical question. In this paper, the possibility that the availability of synthetic horns can directly alter the distribution of  $V_W$  is not considered in detail since it is important as a first step to understand how the market would operate in the absence of such an effect, especially since it is not clear empirically how strong this effect actually is. An examination of this effect, as well as a rigorous analysis of how this effect would impact the efficacy of synthetic horns to reduce wild horn supply, should be pursued in future research.

A central assumption adopted in the paper is that buyers are not able to distinguish between synthetic horns and wild horns (or, equivalently, the cost of doing so is prohibitive). When there is a significant difference in how buyers value wild horns relative to synthetic horns, they have an incentive to invest in some form of testing or information to differentiate between the two products. At the same time, some horn sellers may utilize costly signaling strategies to inform buyers of the type of product they are selling. Note that any mechanism that allows buyers to ascertain the quality of horns for sale—assuming that wild horns are valued more than synthetic horns—essentially adds to the cost of acquiring wild horns relative to a market without any synthetic products. How the efficacy of product verification mechanisms and the cost of testing or signaling affect the impact of synthetic horns on wild horn supply is another topic left for future work.

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# Appendix

Consider the function

$$\tilde{f}(p, Q) \equiv N_B(1 - G(\underline{V}(p))) - (N_S F(p) + Q).$$

**Lemma 14**  $\tilde{f}(p, Q)$  is continuous in  $p$  (and  $Q$ ), with  $\tilde{f}(p, Q) < 0$  for all  $p$  sufficiently large.

**Proof.** Given the continuity of  $F(\cdot)$  (and, hence, the continuity of  $\pi$  in  $p$ ),  $\underline{V}(\cdot)$  is continuous. This implies that  $\tilde{f}(p, Q)$  is continuous in  $p$ . Now, we can make  $\underline{V}(p)$  arbitrarily large by picking  $p$  to be arbitrarily large. This means that, for  $p$  sufficiently high,  $\tilde{f}(p, Q)$  can be made arbitrarily close to  $-(N_S + Q) < 0$ . ■

**Lemma 15** Given  $Q \in [0, N_B]$ , a solution to (2) exists.

**Proof.** Since  $\underline{V}(0) = 0$ , we have  $\tilde{f}(0, Q) = N_B - Q \geq 0$ . In addition,  $\tilde{f}(p, Q) < 0$  for all  $p$  sufficiently large by Lemma 14. Because  $\tilde{f}(p, Q)$  is continuous in  $p$  (Lemma 14 again), the Intermediate Value Theorem then tells us there must be a non-negative  $p$  at which  $\tilde{f}(p, Q) = 0$ . ■

**Example 16** Suppose the distribution of  $c$  among the wild horn sellers is uniform over the interval  $[4, 20]$ , i.e.,  $F(p) = \max\{\min\{\frac{p-4}{16}, 1\}, 0\}$ . Assume that the distribution of  $V_W$  among the buyers is uniform over  $[0, 20]$ , i.e.,  $G(V) = \max\{\min\{\frac{V}{20}, 1\}, 0\}$ , and that  $h(V) = \frac{1}{10}V$ . Let  $N_B = N_S = 100$ . Given  $Q = 5$ , there are three solutions to (2): 1.9, 4.12, and 10.15.

**Lemma 17**  $\tilde{f}(P(Q), Q) = 0$ .

**Proof.** First, note that  $\tilde{f}(P(Q), Q) \not\geq 0$ . If  $\tilde{f}(P(Q), Q) > 0$ , then, since  $\tilde{f}(p, Q) < 0$  for all  $p$  large and  $\tilde{f}(p, Q)$  is continuous in  $p$  (Lemma 14), there would be a  $p > P(Q)$  such that  $\tilde{f}(p, Q) = 0$ , which contradicts  $P(Q)$  being an upper bound for all solutions of the equation  $\tilde{f}(p, Q) = 0$ .

Now, assume that  $\tilde{f}(P(Q), Q) < 0$  (for this to be true, note that  $P(Q)$  has to be strictly positive). By the continuity of  $\tilde{f}(p, Q)$  in  $p$ , there is a neighborhood around  $P(Q)$  such that  $\tilde{f}(p, Q) < 0$  for all  $p < P(Q)$  in this neighborhood. This, however, contradicts  $P(Q)$  being the least upper bound for all solutions of  $\tilde{f}(p, Q) = 0$  (since any  $p < P(Q)$  in this neighborhood must also be an upper bound). ■

**Lemma 18** If  $P(Q)$  is defined as in (4), then it is upper semi-continuous.

**Proof.** To establish upper semi-continuity, we need to show that, for any  $Q^* \geq 0$  and sequence  $\{Q_n\}$  such that  $Q_n \rightarrow Q^*$ ,  $P(Q^*) \geq \lim_{n \rightarrow \infty} p_n$ , where  $p_n \equiv \sup_{m \geq n} P(Q_m)$ .

The proof is by contradiction. Suppose not; suppose to the contrary  $P(Q^*) < \lim_{n \rightarrow \infty} p_n$ . By the definition of  $P(Q^*)$  as given in (4) and the fact that  $\tilde{f}(p, Q^*) < 0$  for all  $p$  sufficiently large (from Lemma 14), we must have  $\tilde{f}(p, Q^*) < 0$  for all  $p > P(Q^*)$ .

Since  $P(Q^*) < \lim_{n \rightarrow \infty} p_n$  by supposition, there must exist  $n$  large enough such that  $P(Q^*) < p_n$ . By the definition of  $p_n$ , we have  $P(Q_m) \leq p_n$  for all  $m \geq n$ . This means there must be an  $m \geq n$  such that  $P(Q_m) \in (P(Q^*), p_n]$  (if not—if there is no such  $m$ —then  $p_n$  cannot be the least upper bound of  $\{P(Q_m)\}_{m \geq n}$ , a contradiction).

The above observations imply that  $\tilde{f}(P(Q_m), Q^*) < 0$ . Since we can pick  $n$  and  $m$  so that  $Q_m$  is arbitrarily close to  $Q^*$ , we obtain  $\tilde{f}(P(Q_m), Q_m) < 0$  by the continuity of  $\tilde{f}(p, Q)$  in  $Q$ . This, however, contradicts  $\tilde{f}(P(Q_m), Q_m) = 0$ , which follows from Lemma 17. ■

#### Proof of Theorem 4

**Proof.** Given any  $Q \in [0, N_B]$ , Lemma 15 tells us that  $\{p \in \mathbb{R}_+ \mid N_B(1 - G(\underline{V}(p))) = N_S F(p) + Q\}$  is non-empty so that  $P(Q)$  is well defined. In addition, Lemma 18 implies that the monopoly's objective function is upper semi-continuous. By an extension of the Weierstrass Extreme Value Theorem [Aliprantis & Border, 2006], the monopoly's profit maximization problem has a solution. Hence, a market equilibrium must exist. ■

**Lemma 19** *If  $h(V) = V$ , then  $P(Q)$  is strictly decreasing.*

**Proof.** First, note that if  $h(V) = V$ , then  $\underline{V}(p) = p$  and  $\tilde{f}(p, Q)$  is decreasing in  $p$ . Take two quantities  $Q_1$  and  $Q_2$  such that  $Q_1 < Q_2$ . Suppose to the contrary that  $P(Q_1) \leq P(Q_2)$ . Since  $\tilde{f}(p, Q)$  is decreasing in  $p$  and is strictly decreasing in  $Q$ , the hypothesis that  $P(Q_1) \leq P(Q_2)$  along with the fact that  $\tilde{f}(P(Q_1), Q_1) = 0$  and  $\tilde{f}(P(Q_2), Q_2) = 0$  (using Lemma 17) yield

$$0 = \tilde{f}(P(Q_2), Q_2) \leq \tilde{f}(P(Q_1), Q_2) < \tilde{f}(P(Q_1), Q_1) = 0.$$

This is a contradiction; therefore,  $P(Q_1) > P(Q_2)$ . ■