

What is the measure of the point set on which

$$f_{xy} = f_{yx}.$$

Problems of this type can be multiplied indefinitely, and a general theory of partial differentiation in the large to cover all such problems is desirable. We have started to construct such a theory. Among the various theorems which we have proved are three which we announce in this note as follows.

**THEOREM 1.** *Let  $f(x, y)$  be of class  $C'$  on an open point set  $H$ . Let the four second partial derivatives*

$$f_{xx} \quad f_{xy} \quad f_{yx} \quad f_{yy}$$

*exist everywhere on  $H$ . Then  $f_{xy} = f_{yx}$  almost everywhere on  $H$ .*

**THEOREM 2.** *Let  $f(x, y)$  be of class  $C'$  on an open point set  $H$ . Let  $f_{xy}$  exist everywhere on  $H$ . Then there exists a point set  $W$  of positive measure on which  $f_{yx}$  exists and equals  $f_{xy}$ .*

**THEOREM 3.** *Let  $f(x, y)$  be of class  $C'$  on the open point set  $H$ . At each point  $P$  of  $H$  let at least one of the two partial derivatives  $f_{xy}, f_{yx}$  exist. Then there exists a set  $W$  of positive measure on which both  $f_{xy}$  and  $f_{yx}$  exist and are equal.\**

\* For certain theorems on second order partial derivatives the reader is referred to the following articles:

Hobson, "Theory of Functions of a Real Variable," Cambridge, pp. 425, 426, 428 (1927).

Carathéodory, "Vorlesungen über reelle Funktionen," Berlin, 650 (1927).

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## EOCENE LAND MAMMALS ON THE PACIFIC COAST

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The Sespe formation comprises a series of sandstones, shales and conglomerate, several thousand feet in thickness, and receives its name from the type locality of its occurrence on Sespe Creek north of the Santa Clara Valley, Ventura County, California. Originally described by Watts,<sup>1</sup> this formation has been recognized as furnishing an important stratigraphic record of the early Tertiary in the southern coast ranges with a position between marine sediments of Eocene age and marine sediments of Miocene age. At a number of localities in southern California deposits having the stratigraphic position and lithologic characteristics of the Sespe

are designated by that name and are regarded as the correlatives of this formation. Prevailing features which have been noted with regard to the Sespe are (1) the presence of red beds, and (2) the absence of a fossil record.

During the past few years a field study has been made of the Sespe with a view to recovering a paleontological record. Although the type section has yielded as yet no material, important fossil evidence has been found at several localities in adjacent regions.<sup>2</sup> It becomes increasingly apparent that the name Sespe, unless definitely restricted to a stratigraphic unit in the type section, loses its significance as a formational

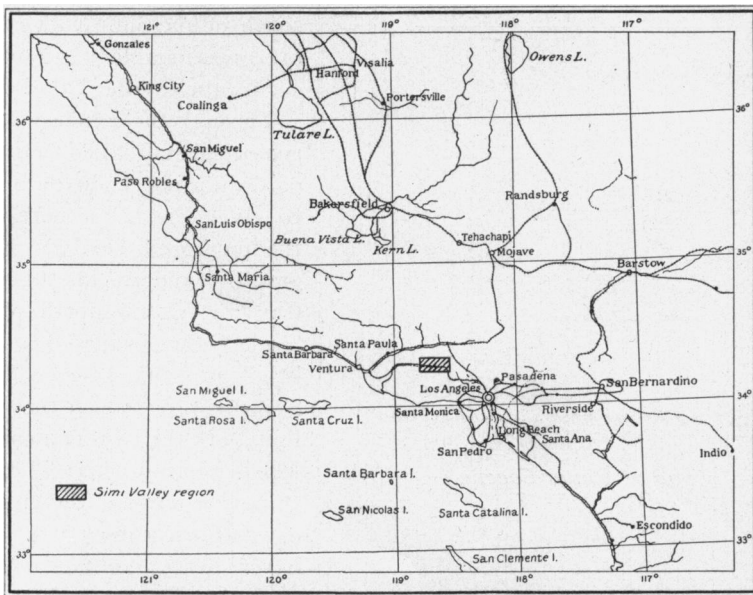


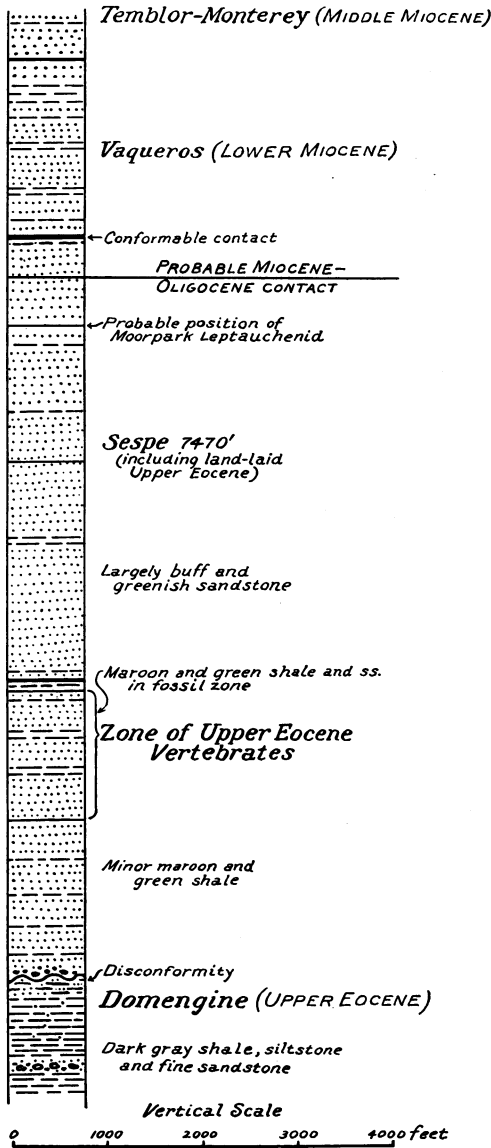
FIGURE 1

Outline map of a portion of southern California showing location of Sespe Eocene mammals in Simi Valley region.

designation and is expressive rather of a distinctive lithologic facies, for in deposits referred to the Sespe are vertebrate faunas which range in age from Eocene to upper Oligocene or lower Miocene.

Among the assemblages now known the oldest is that recorded from deposits exposed south of the Santa Suzanna Mountains and north of the Simi Valley, Ventura County. The region has been studied by several geologists and paleontologists, notably in recent years by W. S. W. Kew, by R. N. Nelson and by B. L. Clark. According to Kew,<sup>3</sup> who has mapped the area, the sequence of Tertiary formations includes marine Eocene, continental Sespe, marine Miocene and Pliocene. A very thick section

of sediments is exposed on the north limb of the Simi anticline immediately to the north of the Simi Valley (Fig. 1).



Courtesy Shell Company of Calif.

FIGURE 2

A portion of the Tertiary stratigraphic column in the Brea Canyon section, north of the Simi Valley, Ventura County, California.

While the Eocene marine strata were regarded originally as belonging to the Tejon or upper division of this period, further study of the invertebrate faunas led Clark to recognize an earlier horizon, namely the Domengine (Upper Eocene). Immediately to the south of the Simi Valley, beds of questionable Tejon age occur above the Domengine.

Through the courtesy of the geological staff, Shell Company of California, I am permitted to give (Fig. 2) a portion of the measured Tertiary stratigraphic column in the Brea Canyon section north of the Simi Valley, as well as the lithologic characters of the Sespe as it occurs on the north limb of the Simi anticline. The deposits are here 7470 feet thick and may be conveniently divided into three parts. The lower Sespe consists of buff-weathering sandstones and conglomeratic sandstones with minor thicknesses of maroon and gray clays. The middle division is represented by maroon and green clays with fine-grained sandstones, alternating with coarse-grained buff and green sandstones. The middle Sespe, in contrast to the lower and upper divisions, is characterized particularly by variegated beds. The upper division consists in the main of

brown-weathering and gray sandstones and conglomerate. An erosion interval separates the lower beds from the marine Eocene (Domengine) while the upper Sespe grades without break into the Vaqueros marine deposits of lower Miocene age.

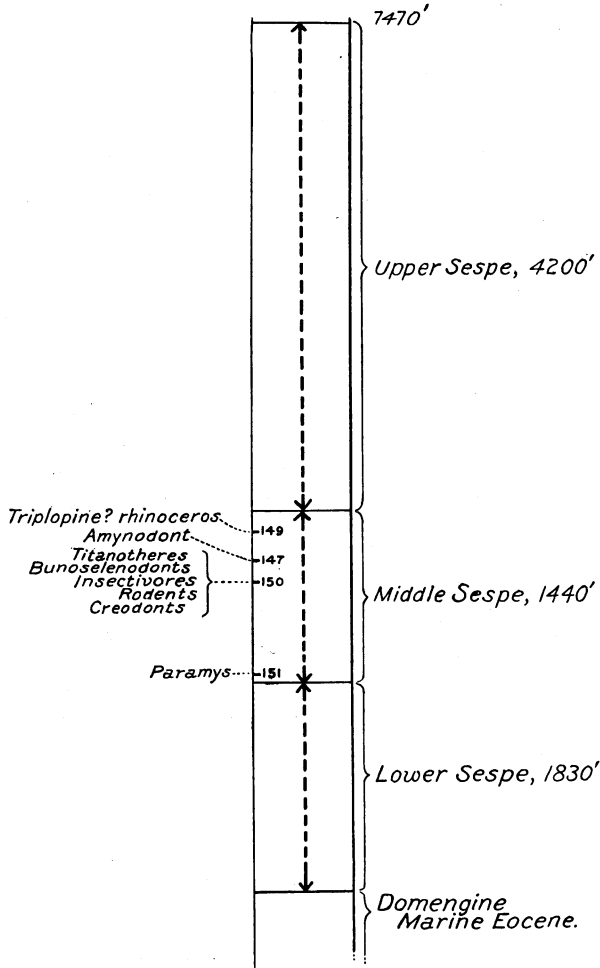


FIGURE 3

Stratigraphic column in Brea Canyon section showing thickness of principal divisions of the Sespe and vertical range of mammalian remains in middle Sespe (Calif. Inst. Tech. Vert. Pale. Localities 151, 150, 147 and 149).

Vertebrate remains have been found at several localities in the maroon and green clays and sandstones of the middle division. Stratigraphically, the occurrences range from approximately 1800 feet to 3000 feet above

the base of the Sespe (Fig. 3). The materials are fragmentary but the fauna as now known comprises at least fourteen different types.

The mammalian assemblage includes titanotheres, rhinoceroses of two types including an advanced amynodont and a small form related to either

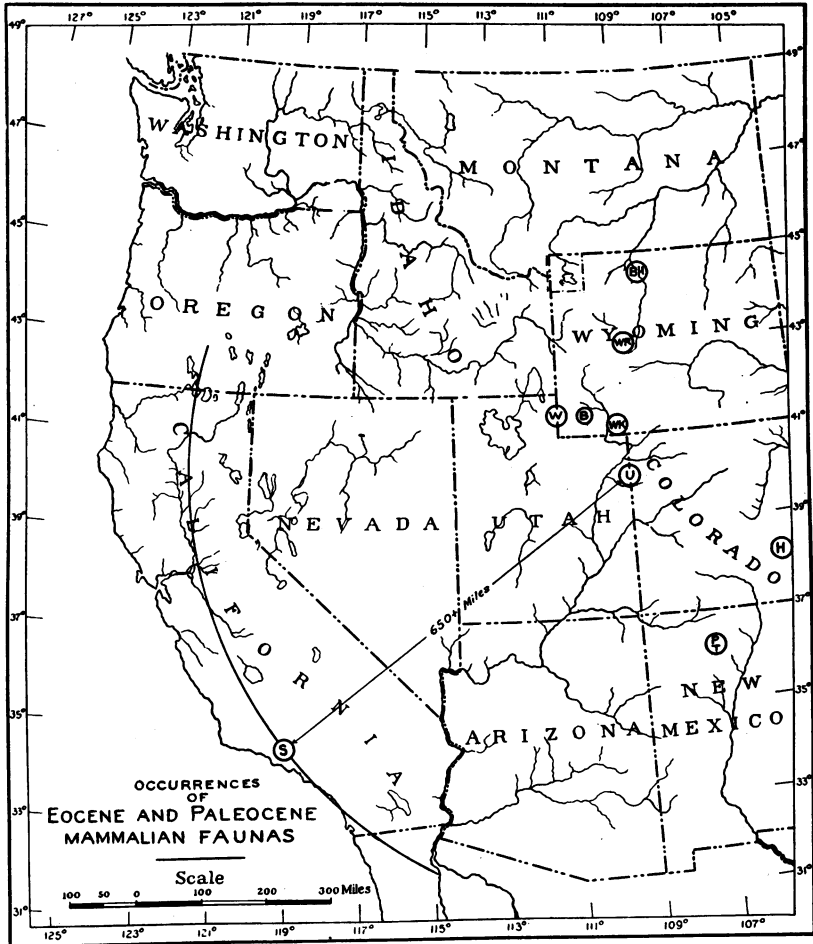


FIGURE 4

Outline map of western states and occurrences of Eocene and Paleocene faunas after Osborn. BH, Big Horn Basin; WR, Wind River; W, Wasatch; B, Bridger; WK, Washakie; U, Uinta; H, Huerfano; PT, Puerco-Torrejon. Newly discovered locality in the Simi Valley region indicated at S.

the triploline or hyracodont group. Among the artiodactyls are *Protylopus* or a closely related form and an ancestral agriocherid. Among the insectivores is a leptictid. Creodonta are represented by at least two

distinct members of the Hyaeodontidae and by a miacid. The rodents include the characteristic Eocene genus *Paramys* and a tiny representative of the cricetine group.

The most nearly related assemblages in time and in space are known to occur in the region of the Rocky Mountains and western Great Plains, 650 miles or more to the east of the Simi localities (Fig. 4). Preliminary study of this fauna, in the light of the known early Tertiary sequence of mammalian assemblages for western North America, suggests an age later than that of the Uinta Eocene (Uinta C) and earlier than that of the White River Oligocene. In this position the relationships appear to be closer to the Eocene than to the Oligocene assemblages. The presence of an amynodont and a triplopine rhinoceros, several distinct types of creodonts and the genus *Paramys* suggests an Eocene age. The titanotheres are known as yet only by isolated teeth. A number of incisors belonging to these types are characterized by rounded, reduced crowns, and in this respect the Simi forms are evidently more advanced than the Eocene genera *Dolichorhinus*, *Telmatherium* and *Protitanotherium*, and are more like types found in the Titanotherium zone of the White River. Among the creodonts is the genus *Hyaenodon*, generally regarded as characteristic of Oligocene faunas in North America. It is noteworthy, however, that the relatively large number of carnivores found at this stage comprises at least two families of the Creodonta with no representatives of the *Carnivora vera*.

A fauna exhibiting a similar stage of evolution has been described recently by Peterson<sup>4</sup> from deposits overlying the Upper Uinta (Uinta C) of the Uinta Basin. On the basis of relatively advanced types of titanotheres, provisionally referred to the genus *Teleodus*, the advanced character of the horses, tapiroids and cameloids, and the presence of a true *Hyaenodon*, Peterson assigns this stage to the basal Oligocene, designating it the Duchesne. Possibly the Sespe fauna of the Simi region, which resembles rather closely the Duchesne assemblage in stage of development, should be assigned likewise to the basal Oligocene. However, the number of distinctive Eocene types and the position of the fauna in the stratigraphic series above beds which do not represent apparently the highest Eocene marine strata as recognized in the Coast Ranges of California, have led me to regard the assemblage, at least for the present, as belonging to the uppermost Eocene.

<sup>1</sup> Watts, W. L., *Calif. State Min. Bur. Bull.*, 11, 22-38 (1897).

<sup>2</sup> Stock, C., *Carnegie Inst. Wash. Publ.* 404, Art. III (1930).

<sup>3</sup> Kew, W. S. W., *U. S. G. S. Bull.* 691-M (1919).

<sup>4</sup> Peterson, O. A., *Ann. Carnegie Mus.*, 21, 61-78 (1931).